

Ring Accelerator Physics for Non Accelerator Specialists

XFEL Research & Development Division

RIKEN SPring-8 Center

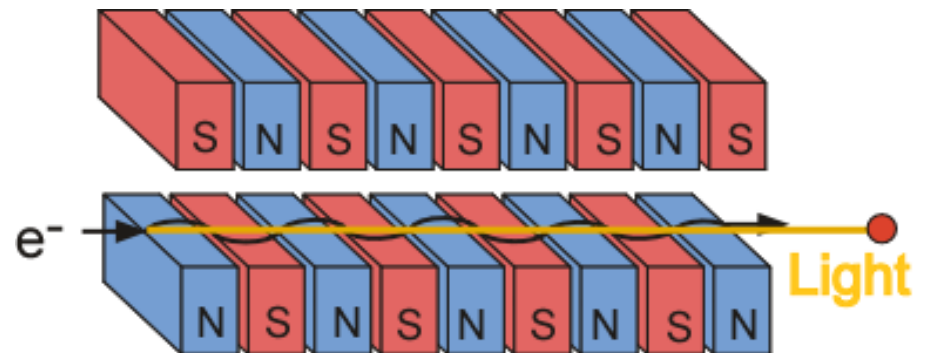
Hitoshi Tanaka

Outline

1. Light source properties vs electron beam performance
2. Stochasticity of photoemission
3. Distribution of circulating electron beam
4. Approach to coherent X-rays

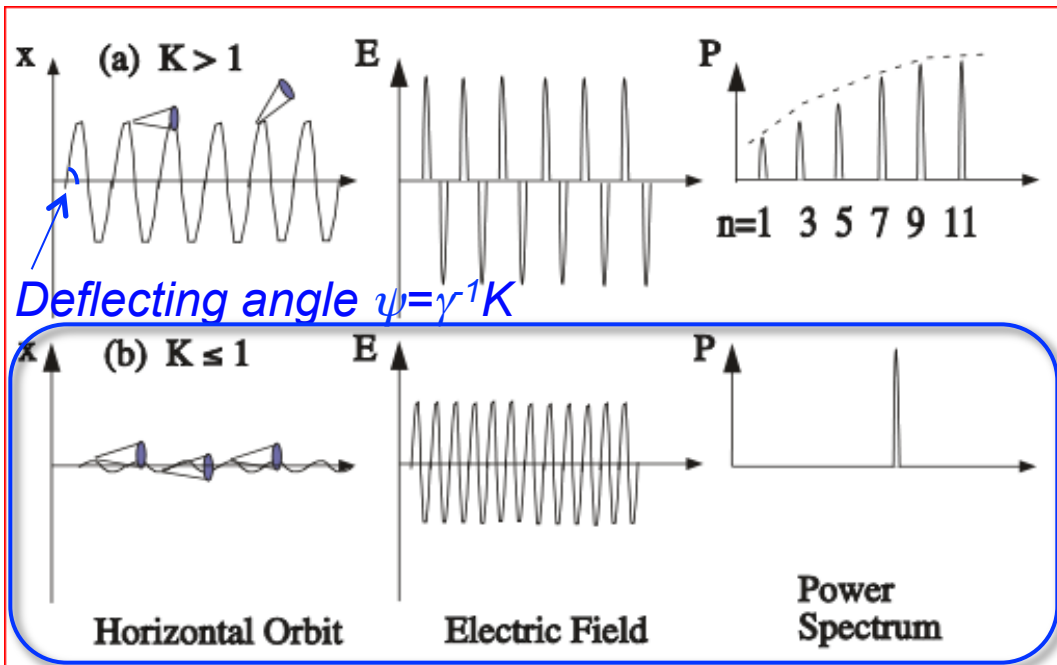
1. Light source properties vs electron beam performance(1)

Main devices to supply SR to users are “undulators”, which are installed in magnet-free straight sections in 3rd-generation SR sources.



1. Light source properties vs electron beam performance(2)

In an undulator, radiation from a single electron at each undulation interferes with each other.



Spectral narrowing by resonant condition for a single electron is

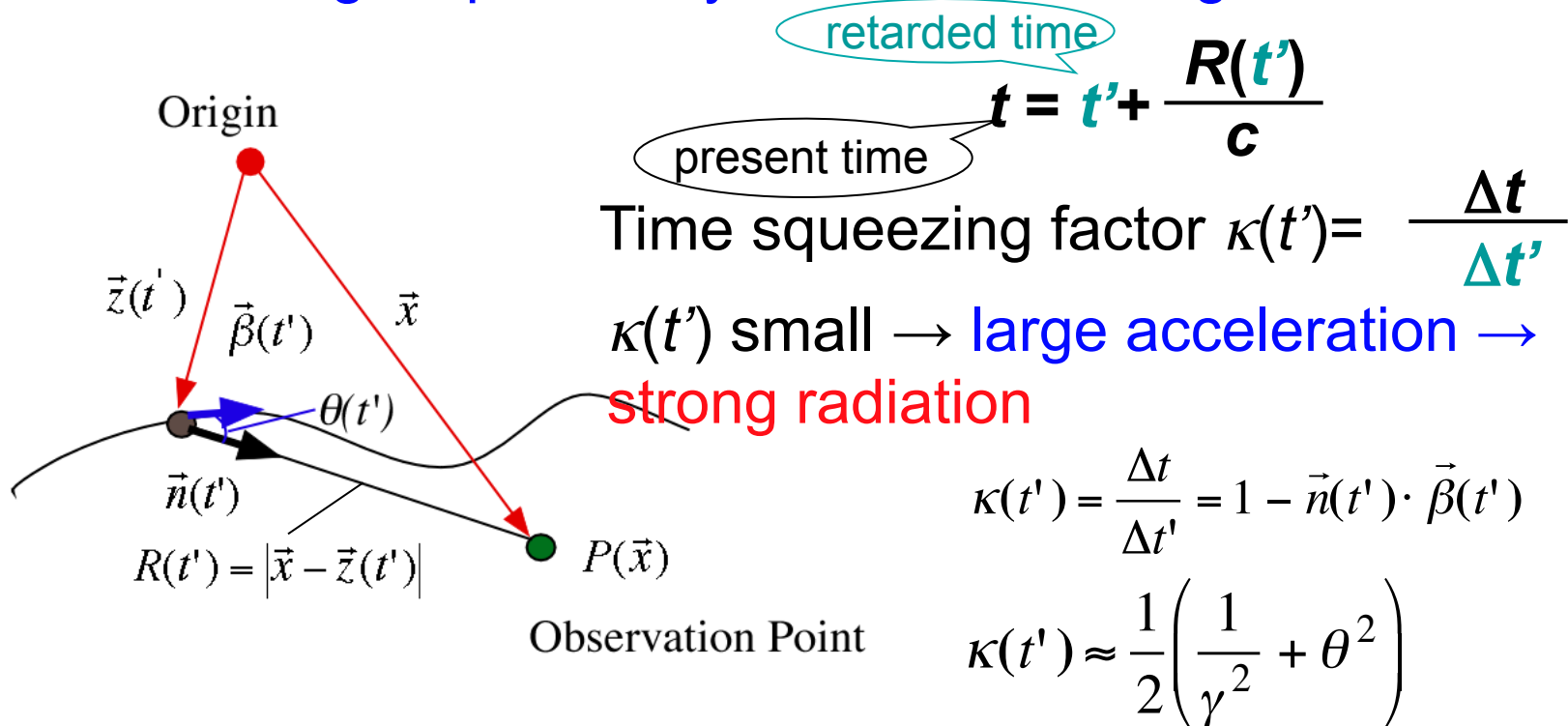
$$\lambda_1 = \frac{\lambda_u}{2\gamma^2} (1 + K^2/2 + \gamma^2 \theta^2)$$

where K and γ are a deflecting parameter and relative energy.

1. Light source properties vs electron beam performance(3)

Q: Why does the radiation concentrate within the γ cone ?

A: The electron can run after the emitted light with the almost light speed only in this limited angle.

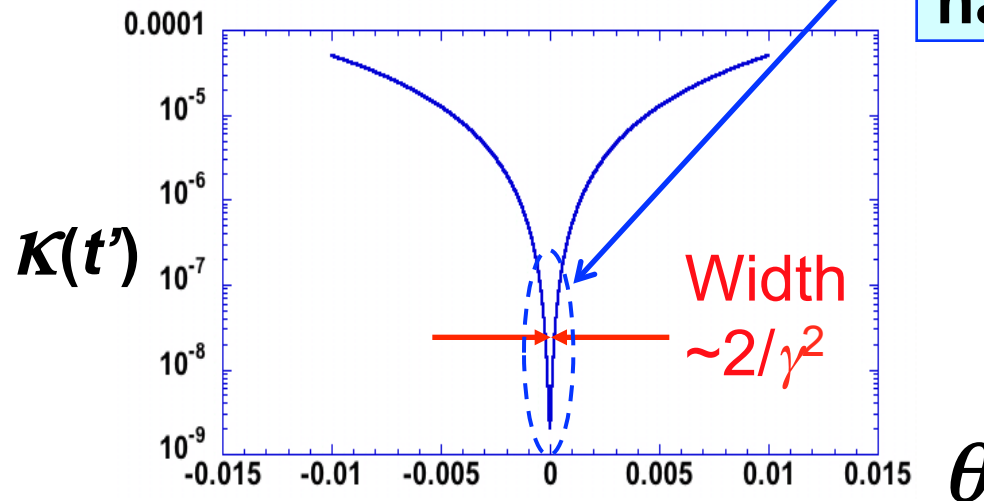


1. Light source properties vs electron beam performance(4)

Q: Why does the radiation concentrate within the γ cone ?

A: The electron runs after the emitted light with the almost light speed only in this limited angle.

$$K(t') \approx \frac{1}{2} \left(\frac{1}{\gamma^2} + \theta^2 \right) \quad \theta < 1/\gamma \rightarrow K \sim 1/\gamma^2$$

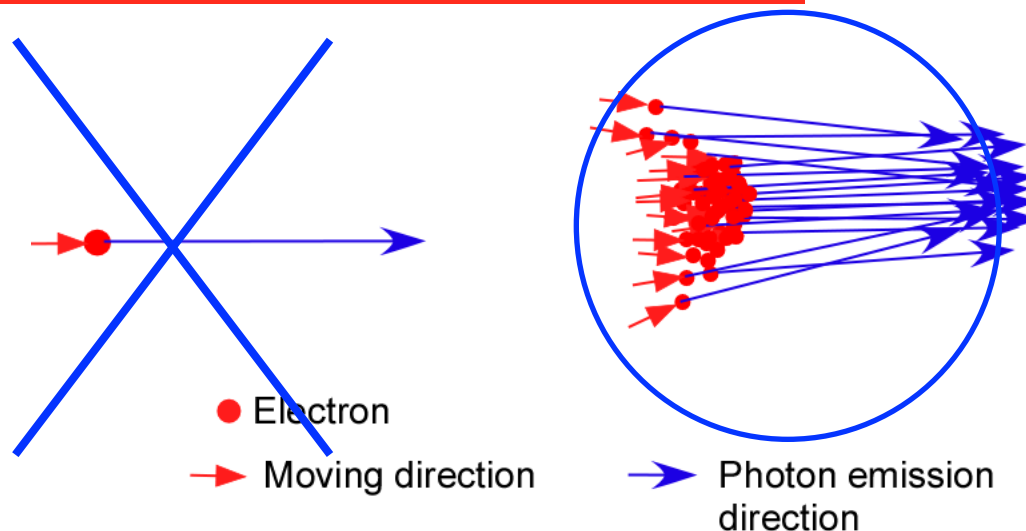


Intense radiation is emitted in this narrow angle

1. Light source properties vs electron beam performance(5)

If electron beam were point-like without spatial divergence and , all electrons could have no angular divergence, radiations were **coherent** !

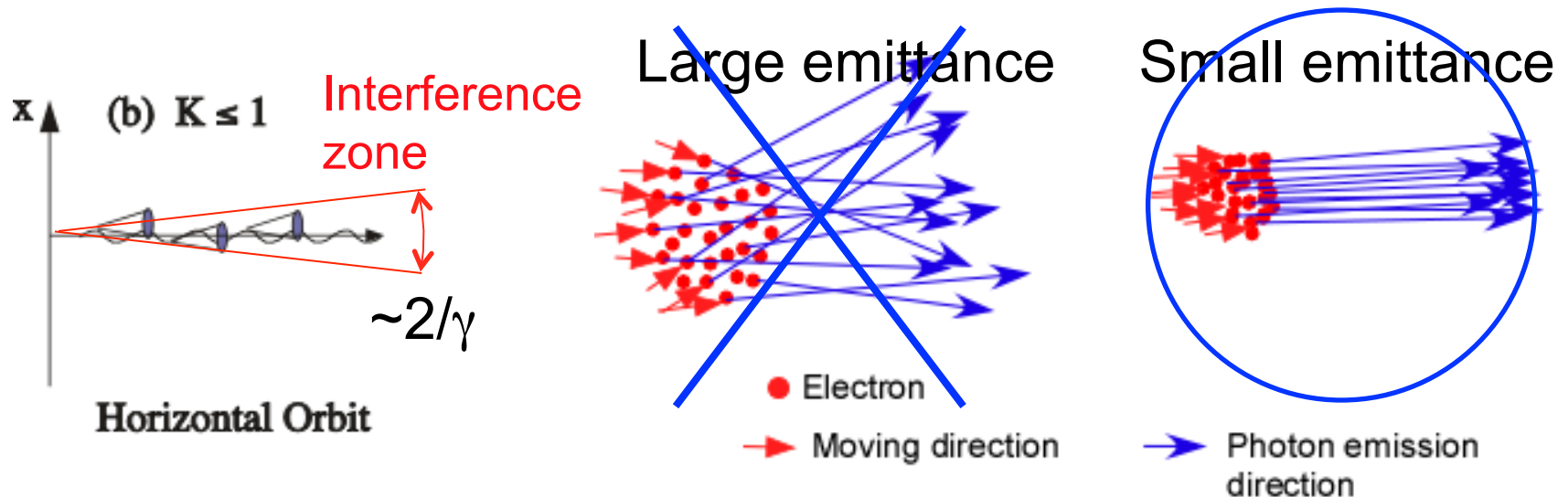
Real radiation properties are obtained by convoluting radiations from all N electrons



In the real world N electrons are distributed in a phase space, never degenerate on the same point.

1. Light source properties vs electron beam performance(6)

In order to obtain the same resonance condition over most of circulating electrons distributed, sufficiently **small beam emittance** is required.



1. Light source properties vs electron beam performance(7)

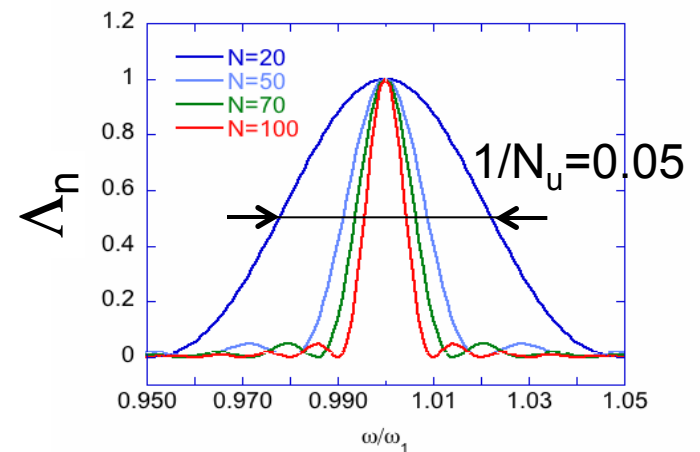
In order to obtain the same resonance condition over most of circulating electrons distributed, sufficiently **small beam energy spread** is required.

$$\lambda_1 = \frac{\lambda_u}{2(\gamma + \Delta\gamma)^2} (1 + K^2/2 + \gamma^2 \theta^2)$$

$$\Delta\lambda_1 \sim 2\lambda_1 <\Delta\gamma/\gamma>$$

Spectral broadening by the energy spread is much less than intrinsic broadening $\sim 1/N_u$.

N_u : undulator period number



1. Light source properties vs electron beam performance(8)

$$B \propto \frac{I_b \cdot N_u^2 \cdot \gamma^2 \cdot F_n(K, \delta)}{\sigma_x \sigma_{x'} \sigma_y \sigma_{y'} (\sim \epsilon_x \cdot \epsilon_y)},$$

B: Brilliance (phs/sec/mm²/mrad²/100mA)

I_b: Beam current (mA)

N_u: Undulator period number

σ_x, σ_y: Horizontal and vertical beam sizes (m)

σ_{x'}, σ_{y'}: Horizontal and vertical angular divergence (rad)

δ: Beam energy spread

K :Deflection parameter

1. Light source properties vs electron beam performance(9)

$$\sigma_x = \sqrt{\sigma_p^2 + \beta_x \epsilon_x + \eta_x^2 \delta^2}, \quad \sigma_{x'} = \sqrt{\sigma_p'^2 + \gamma_x \epsilon_x + \eta'_x{}^2 \delta^2},$$

$$\sigma_y = \sqrt{\sigma_p^2 + \beta_y \epsilon_y + \eta_y^2 \delta^2}, \quad \sigma_{y'} = \sqrt{\sigma_p'^2 + \gamma_y \epsilon_y + \eta'_y{}^2 \delta^2},$$

$$\alpha_{x,y} = \frac{-d\beta_{x,y}}{ds}, \quad \gamma_{x,y} = \frac{1 + \alpha_{x,y}^2}{\beta_{x,y}}, \quad \eta'_{x,y} = \frac{d\eta_{x,y}}{ds}.$$

Photon

**Electron
emittance**

**Electron
energy
spread**

ϵ_x, ϵ_y : Horizontal and vertical emittance (m•rad)

$\beta_{x,y}$: Horizontal and vertical betatron functions at ID

$\eta_{x,y}$: Horizontal and vertical dispersion functions at ID

$\sigma_p, \sigma_{p'}$: Spatial and angular divergence of photon beam

1. Light source properties vs electron beam performance
- 2. Stochasticity of photoemission**
3. Distribution of circulating electron beam
4. Extra: Approach to coherent X-rays

2. Stochasticity of photoemission(1)

The photo-emission process is **not continuous** but **quantized**. So, the emission position, the number of the emission photons, and the emission photon energy have **fluctuations**.

This stochasticity (random fluctuation) causes finite spread of the circulating electron beam in the 6D phase space. The density distribution is generally Gaussian due to the central limit theorem.



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2. Stochasticity of photoemission(2)

A relativistic electron accelerated in a magnetic field will radiate electromagnetic energy at a rate which is proportional to the square of the accelerating force.

<Averaged radiation power>

$$P = \frac{2}{3} \frac{r_e c}{(m_0 c^2)^3} E^2 F_{\perp}^2 = \frac{2}{3} \frac{r_e c}{(m_0 c^2)^3} \frac{E^4}{\rho^2} .$$

classical electron radius

<Averaged radiation energy per turn>

$$U = \int P dt = \int P \frac{d\ell}{c},$$

Averaged properties
are smooth !

$$U(\text{keV}) = \frac{88.5 E^4 (\text{GeV})}{\rho(\text{m})} \text{ for the case with the constant } \rho.$$

2. Stochasticity of photoemission(3)

In the quantized radiation process, the integrated parameter, energy loss per unit time also fluctuates. Magnitude of the fluctuation can be defined by a mean square.

<Mean square of energy loss fluctuation per unit time>

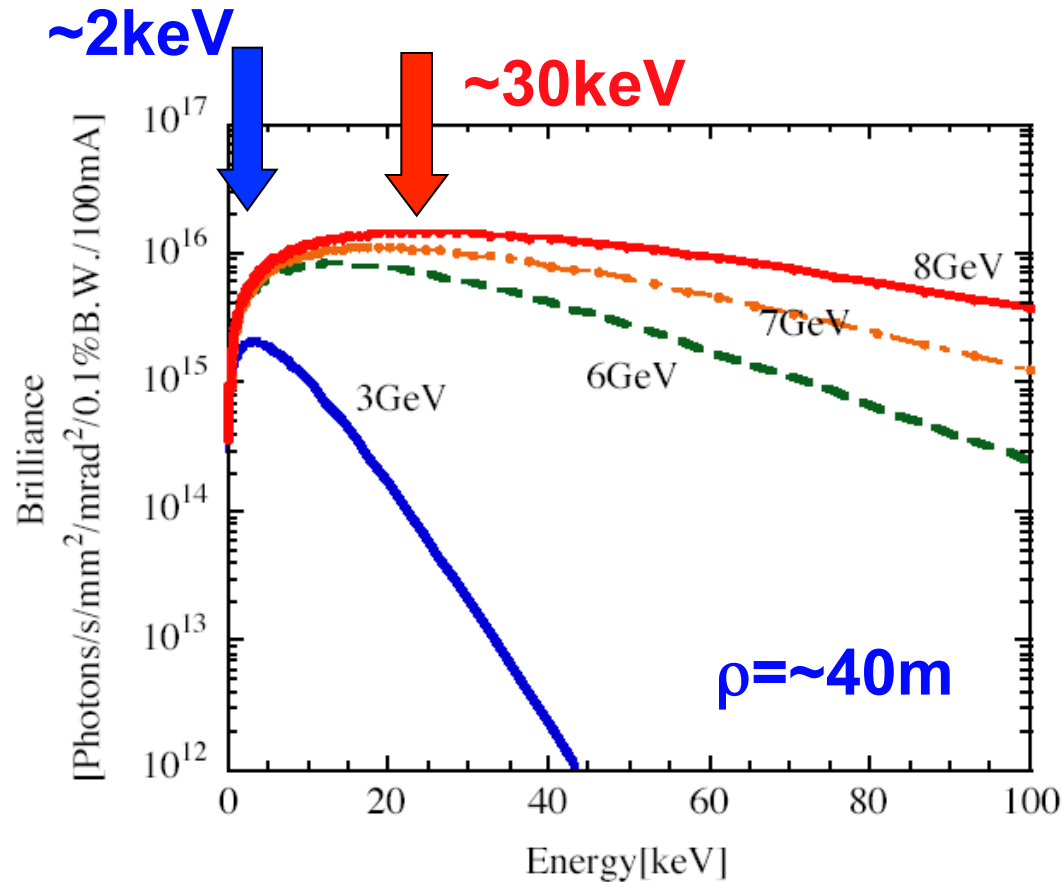
$$\langle Nu^2 \rangle = \frac{55}{24\sqrt{3}} \frac{U}{T} u_c, \quad u_c = \frac{3}{2} \frac{\hbar c}{\rho} \gamma^3.$$

u_c : critical photon energy $\sim 3.2\langle u \rangle$

<Averaged photo-emission rate>

$$\langle N \rangle (\text{phs/s}) \sim 3.2P / u_c$$

2. Stochasticity of photoemission(4)



u_c represents the photon energy at the peak brilliance of the BM radiation.

2. Stochasticity of photoemission(5)

Let's estimate the parameters!

<Case-1 E=1 GeV, $\rho=5\text{m}$ >

$$U(\text{keV}) = \frac{88.5 \times 10^4}{5} = 17.7, \quad P(\text{keV/s}) = \frac{U}{T=2\pi \times 5/c} = 1.7 \times 10^8$$

$$u_c (\text{keV}) = \frac{3}{2} \frac{6.85 \times 10^{-19} \times 2.998 \times 10^8 \times 1957^3}{5} = \mathbf{0.44}$$

$$\langle \text{Nu}^2 \rangle (\text{keV}^2/\text{s}) = \frac{55}{24\sqrt{3}} 1.7 \times 10^8 \times 0.44 = 0.99 \times 10^8$$

$$\langle N \rangle (\text{photons/s}) \sim 3.2 \times 1.7 \times 10^8 / 0.44 = \mathbf{1.2 \times 10^9}$$

2. Stochasticity of photoemission(6)

SPring-8 parameter is the next example.

<Case-2 E=8 GeV, $\rho=40\text{m}$ >

$$U(\text{keV}) = \frac{88.5 \times 8^4}{40} = 9.1 \times 10^3, \quad P(\text{keV/s}) = \frac{U}{T=2\pi \times 40/c} = 1.1 \times 10^{10}$$

$$u_c (\text{keV}) = \frac{3}{2} \frac{6.85 \times 10^{-19} \times 2.998 \times 10^8 \times 15656^3}{40} = \mathbf{29.6}$$

$$\langle \text{Nu}^2 \rangle (\text{keV}^2/\text{s}) = \frac{55}{24\sqrt{3}} 1.1 \times 10^{10} \times 29.6 = 4.31 \times 10^{11}$$

$$\langle N \rangle (\text{photons/s}) \sim 3.2 \times 1.1 \times 10^{10} / 29.6 = \mathbf{1.2 \times 10^9}$$

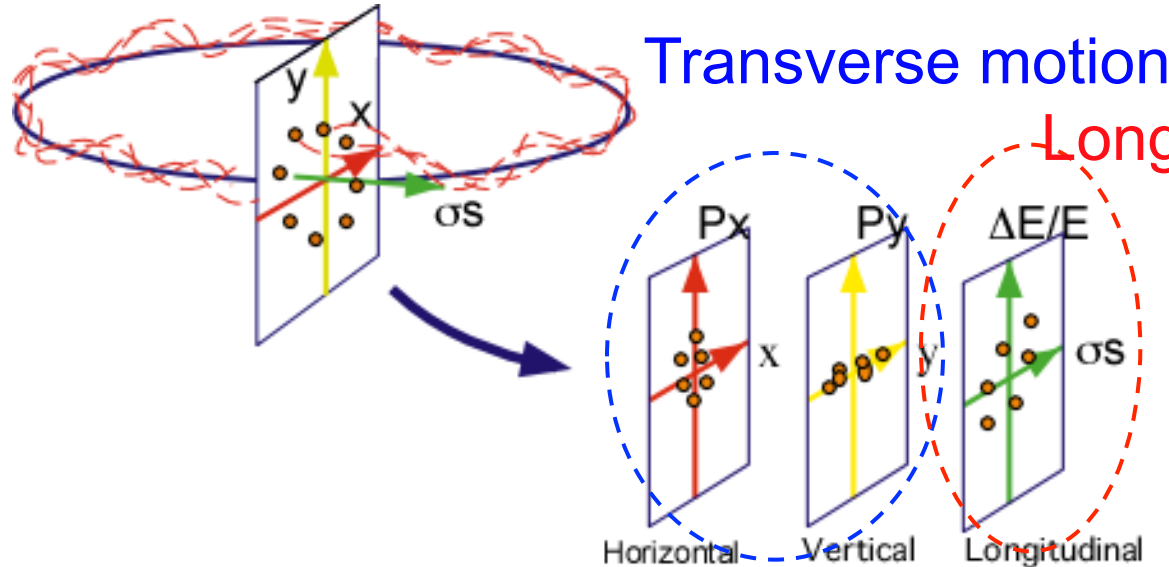
1. Light source properties vs electron beam performance
2. Stochasticity of photoemission
3. **Distribution of circulating electron beam**
4. Extra: Approach to coherent X-rays

3. Distribution of circulating electron beam(1)

6D-phase space volume of a single electron, ϵ_{s6} comprises of canonical variables $(x, px(x'), y, py(y'), t, ps(\Delta E/E))$.

In an ideal case, 6D-phase space volume can be written by the product of areas of the three orthogonal 2D spaces ϵ_{sz} .

$z=x,y,s$



$$\epsilon_{s6} = \epsilon_{sx} \epsilon_{sy} \epsilon_{ss}$$

These are invariants of the motion for a **conservative system**.

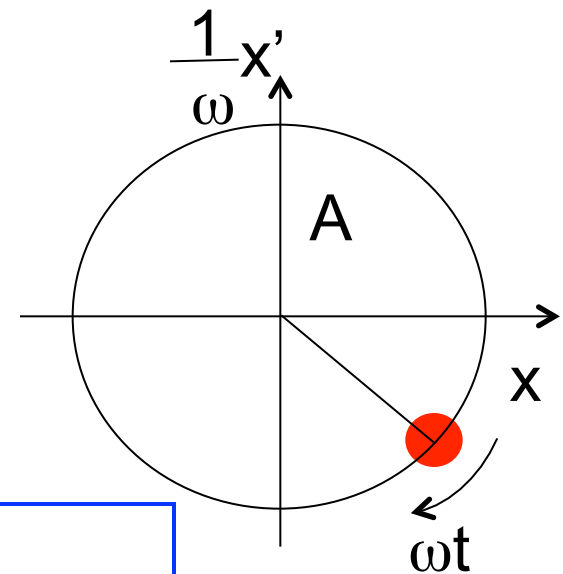
3. Distribution of circulating electron beam(2)

You can understand the **relation between the 2D phase space and the emittance** by using a simple harmonic oscillator.

$$x = A \cdot \cos(\omega t + \phi_0)$$
$$\frac{dx}{dt} = x' = -A \cdot \omega \sin(\omega t + \phi_0)$$
$$\longrightarrow \frac{1}{\omega} \frac{dx}{dt} = -A \cdot \sin(\omega t + \phi_0)$$

$$\text{Action} = A, \text{ Angle} = \phi = \omega t + \phi_0,$$

$$\text{Invariant} = x^2 + (x'/\omega)^2 = A^2 = \frac{\text{circle area}}{\pi} = \epsilon_{sx}$$

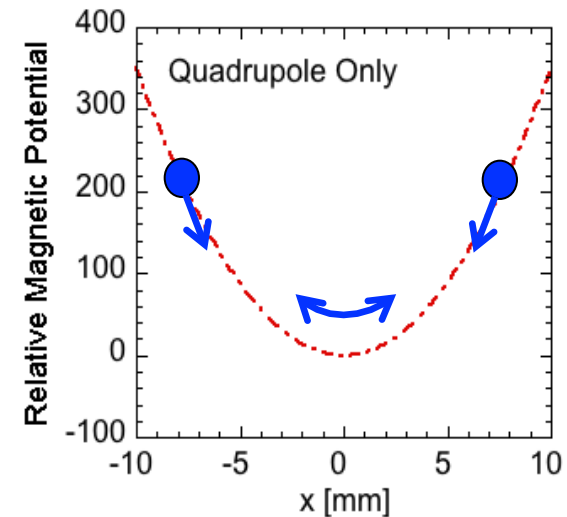


3. Distribution of circulating electron beam(3)

We need **potential wells to stabilize three orthogonal oscillation modes** to keep electron beam in a storage ring.

Quadrupole magnets generate the adequate potential wells for two transversal oscillation modes, which are called **betatron oscillations** in the horizontal and vertical planes.

RF acceleration electric field generates the adequate potential well for longitudinal oscillation mode, which is called a synchrotron oscillation.



3. Distribution of circulating electron beam(4)

SR light properties reflects the 3×2D phase space distribution of circulating electrons.

We use the following **three ensemble-averaged emittances** to express beam distribution in three orthogonal phase spaces.

$$\begin{aligned}\langle \varepsilon_{s6} \rangle &= \langle \varepsilon_{sx} \rangle \langle \varepsilon_{sy} \rangle \langle \varepsilon_{ss} \rangle \\ &= \varepsilon_x \varepsilon_y \varepsilon_s,\end{aligned}$$

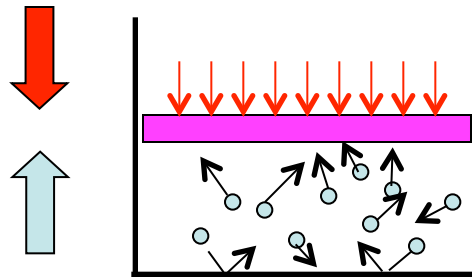
ε_x : horizontal emittance (m rad)

ε_y : vertical emittance (m rad)

ε_s : longitudinal emittance (m rad)

3. Distribution of circulating electron beam(5)

Width of the Gaussian distribution of N circulating electrons is determined by the **dynamical equilibrium between the radiation excitation and damping.**



3. Distribution of circulating electron beam(6)

Remember the invariant of a harmonic oscillator.

$$\frac{d\langle A^2 \rangle}{dt} = \frac{d\varepsilon_z}{dt} \quad z = x, y, s.$$

Equilibrium condition:

$$\frac{d\langle A^2 \rangle}{dt} = \lim_{\Delta t \rightarrow 0} \frac{\langle (A + \Delta A)^2 - A^2 \rangle}{\Delta t} = \lim_{\Delta t \rightarrow 0} 2 \frac{\langle A \Delta A \rangle}{\Delta t} + \lim_{\Delta t \rightarrow 0} \frac{\langle \Delta A^2 \rangle}{\Delta t} = 0$$

Damping term

Averaged energy dissipation

Excitation term

Quantum effect

3. Distribution of circulating electron beam(7)

Energy spread $\sigma_{\Delta E}$

$$\lim_{\Delta t \rightarrow 0} 2 \frac{\langle A \Delta A \rangle}{\Delta t} = -2 \frac{\langle A^2 \rangle}{\tau_\varepsilon}, \quad \tau_\varepsilon = \frac{E \times T}{U},$$

$$\lim_{\Delta t \rightarrow 0} \frac{\langle \Delta A^2 \rangle}{\Delta t} = \langle N u^2 \rangle = \frac{55}{24\sqrt{3}} \frac{U}{T} u_c.$$

Since $\sigma_{\Delta E}$ is **not the emittance**, phase average factor should be considered, $\sigma_{\Delta E}^2 = \frac{\langle A^2 \rangle}{2}$,

$$\sigma_{\Delta E} = \sqrt{\frac{55}{96\sqrt{3}} E \times u_c}, \quad \sigma_{\Delta E/E} = \sqrt{\frac{55}{96\sqrt{3}} \frac{u_c}{E}}.$$

3. Distribution of circulating electron beam(8)

Bunch length σ_τ

σ_τ has two components;

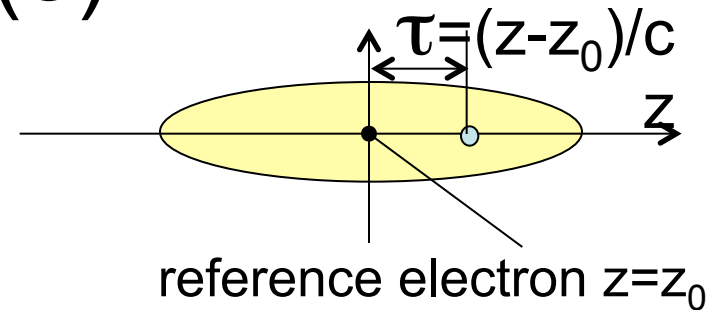
$\sigma_{\tau 1}$ = time spread by energy spread

$\sigma_{\tau 2}$ = time spread by stochastic photoemission

$\sigma_{\tau 1} \gg \sigma_{\tau 2}$

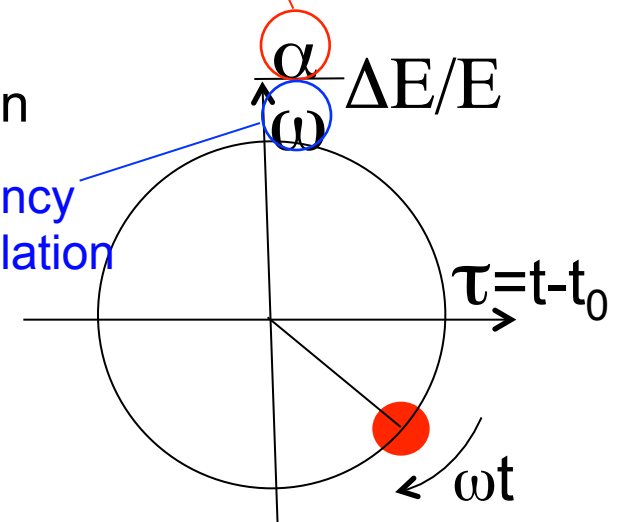
$$\sigma_\tau = \frac{\alpha}{\omega} \sigma_{\Delta E/E} = \sim 10 \text{ psec} @ \text{SPring-8}$$

10^{-4}
 10^{-3}
 10^4



dilation factor

angular frequency
of energy oscillation



Phase space of synchrotron
(energy) oscillation

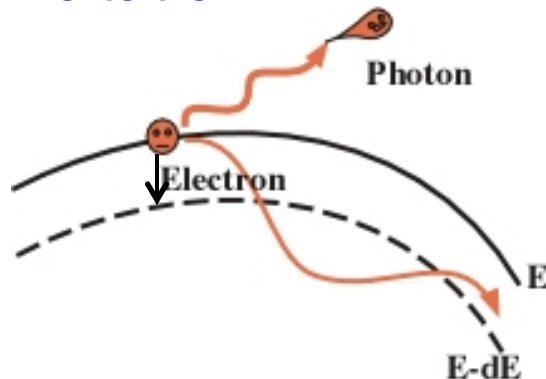
3. Distribution of circulating electron beam(9)

Horizontal emittance ϵ_x

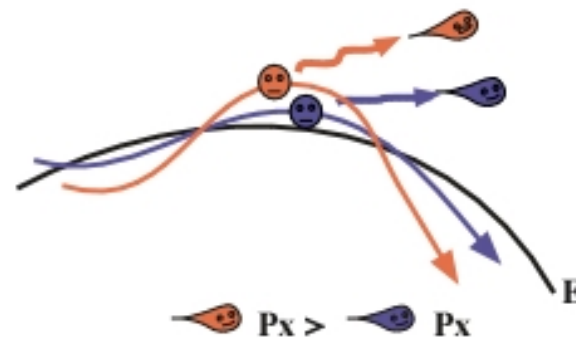
Excitation: due to the discrete energy jump + energy dispersion

Damping: due to the decrease of transverse momentum by the photoemission + acceleration along the running direction

Excitation



Damping



3. Distribution of circulating electron beam(10)

For the typical magnetic lattice structure (Chasman Green: CG) based storage ring, the horizontal minimum emittance is written by

$$\begin{aligned}\epsilon_{x \text{ min}} @\text{general} &\sim \frac{1}{2} \epsilon_{x \text{ min}} @\text{achromat} \\ &= \frac{C_q \gamma^2}{8\sqrt{15} J_x} \theta_b^3\end{aligned}$$

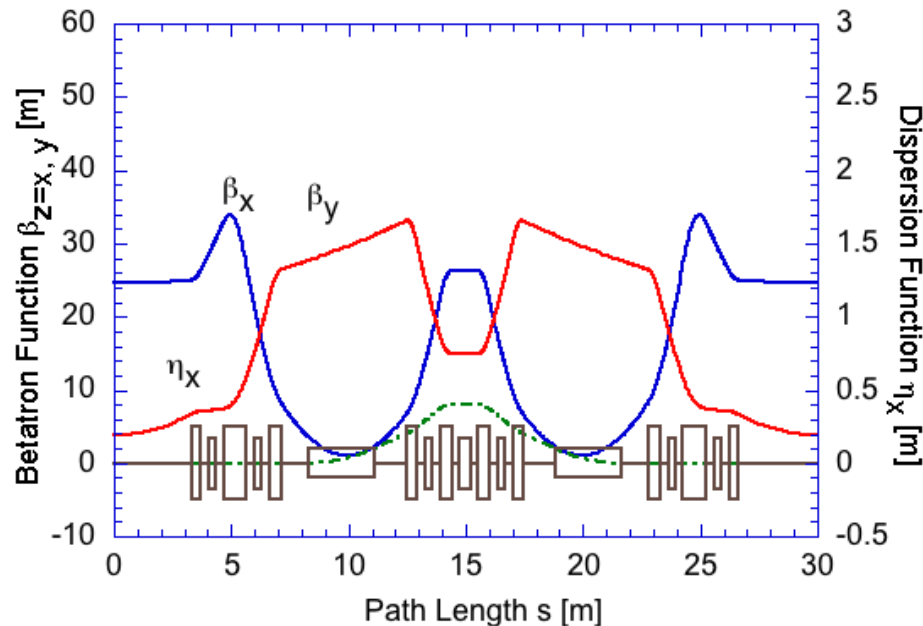
C_q : Quantum constant 3.832×10^{-13} (m)

θ_b : Deflection angle of a single bending magnet (rad)

J_x : Horizontal damping partition number ~ 1

3. Distribution of circulating electron beam(11)

Chasman-Green (CG) lattice is the most popular magnet cell structure for a low emittance SR source, where a achromatic arc is composed of a pair of bending magnets.



2 bends: CG or CBA
3 bends: TBA
4 bends: QBA

$$\epsilon_{x \min} \propto \theta_b^3$$

θ_b smaller
with same
cell No.

3. Distribution of circulating electron beam(12)

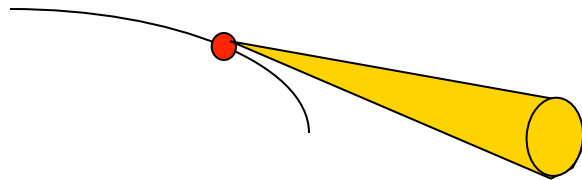
Vertical emittance ϵ_y

ϵ_y has two components;

ϵ_{y1} = vertical emittance by stochastic photoemission

ϵ_{y2} = vertical emittance by HV coupling

Usually, $\epsilon_{y1} \ll \epsilon_{y2}$



The Angular divergence
In the vertical plane is $\sim 1/\gamma$

1-GeV storage ring

$\gamma = 1000/0.511 \sim 2000$

$1/\gamma \sim 5 \times 10^{-4}$

Principally, $\epsilon_y = 1/1000 \epsilon_x$
is possible

3. Distribution of circulating electron beam(13)

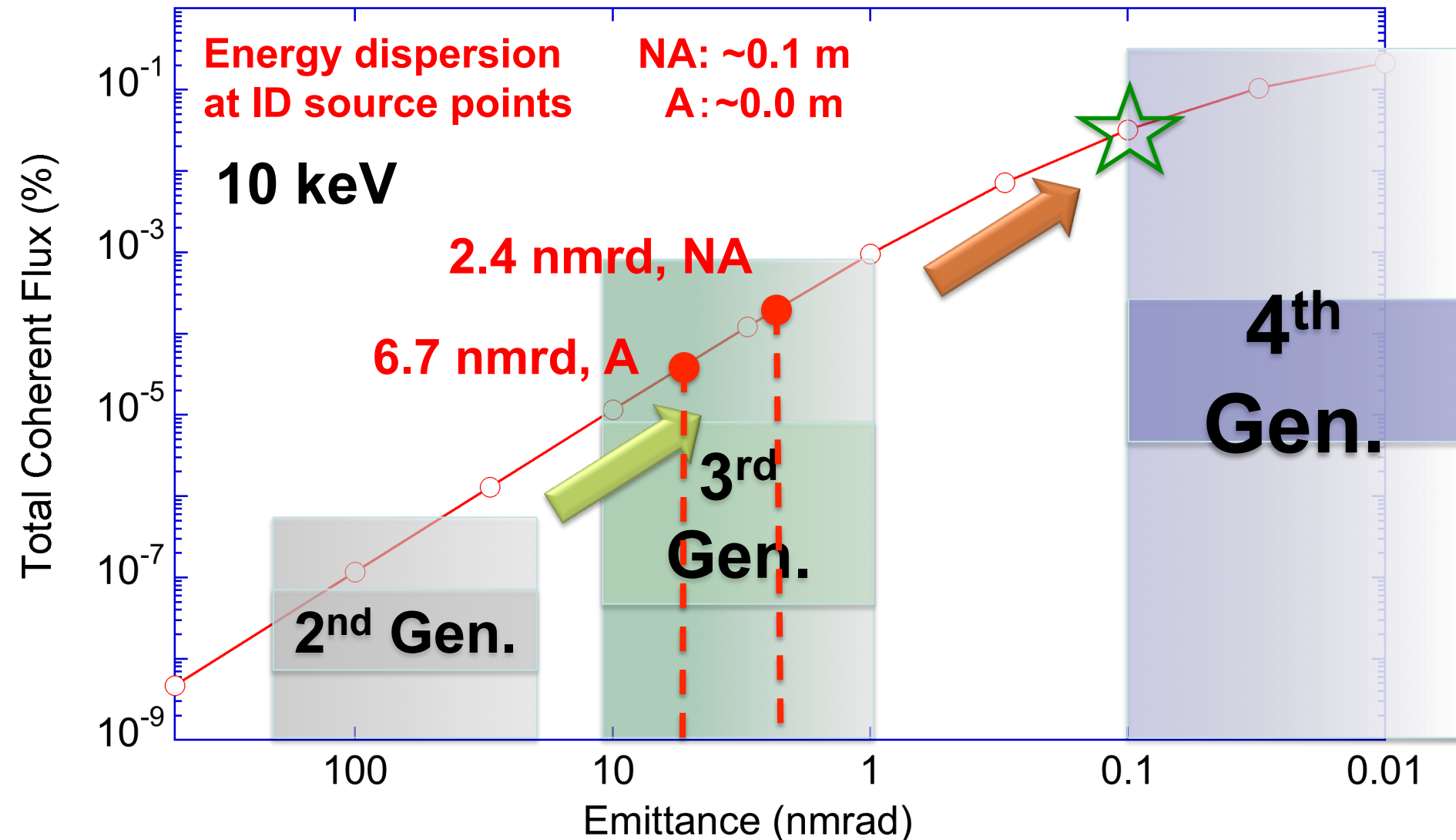
Vertical emittance is determined by magnetic error components mixing the horizontal and vertical betatron oscillation. The main sources are vertical misalignments of sextupole magnets and rotational errors of quadrupole magnets.

The effect of these error fields can be corrected to **0.1 % level** by the combination of beam response analysis and skew quadrupole corrector magnets.

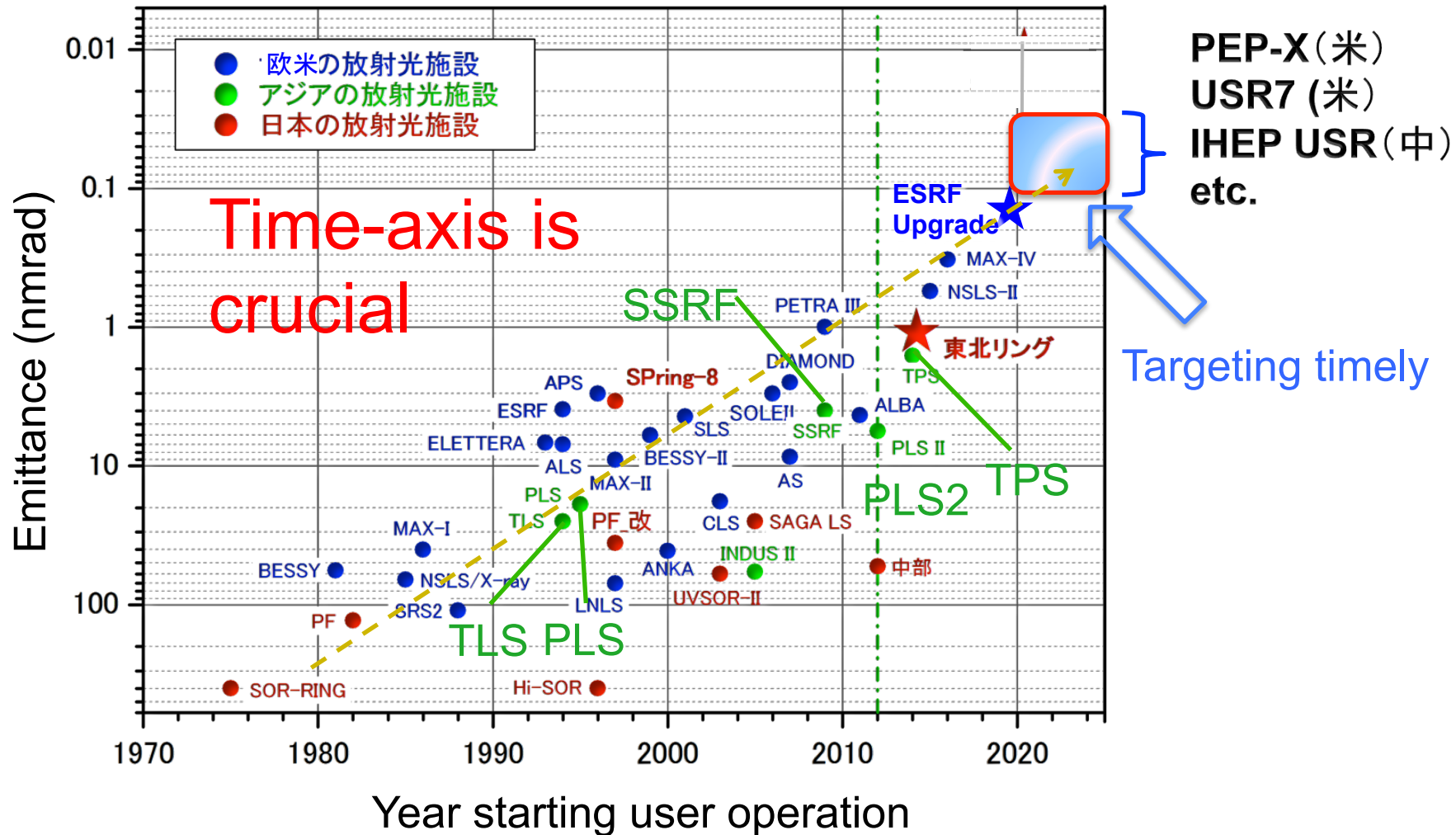
One-dimensional diffraction limited X-ray beam is now available

1. Light source properties vs electron beam performance
2. Stochasticity of photoemission
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4. Approach to coherent X-rays

4. Approach to coherent X-rays (1)



4. Approach to coherent X-rays (2)



4. Approach to coherent X-rays (3)

Equation of natural emittance:

$$\varepsilon_{nat} = C_q \frac{\gamma^2 \langle H/\rho^3 \rangle}{J_x \langle 1/\rho^2 \rangle} \propto \frac{\gamma^2 \theta^3}{J_x}$$

Conventional reduction scheme:

1. Reduction of bending angle (θ) by increasing the number of bending magnets

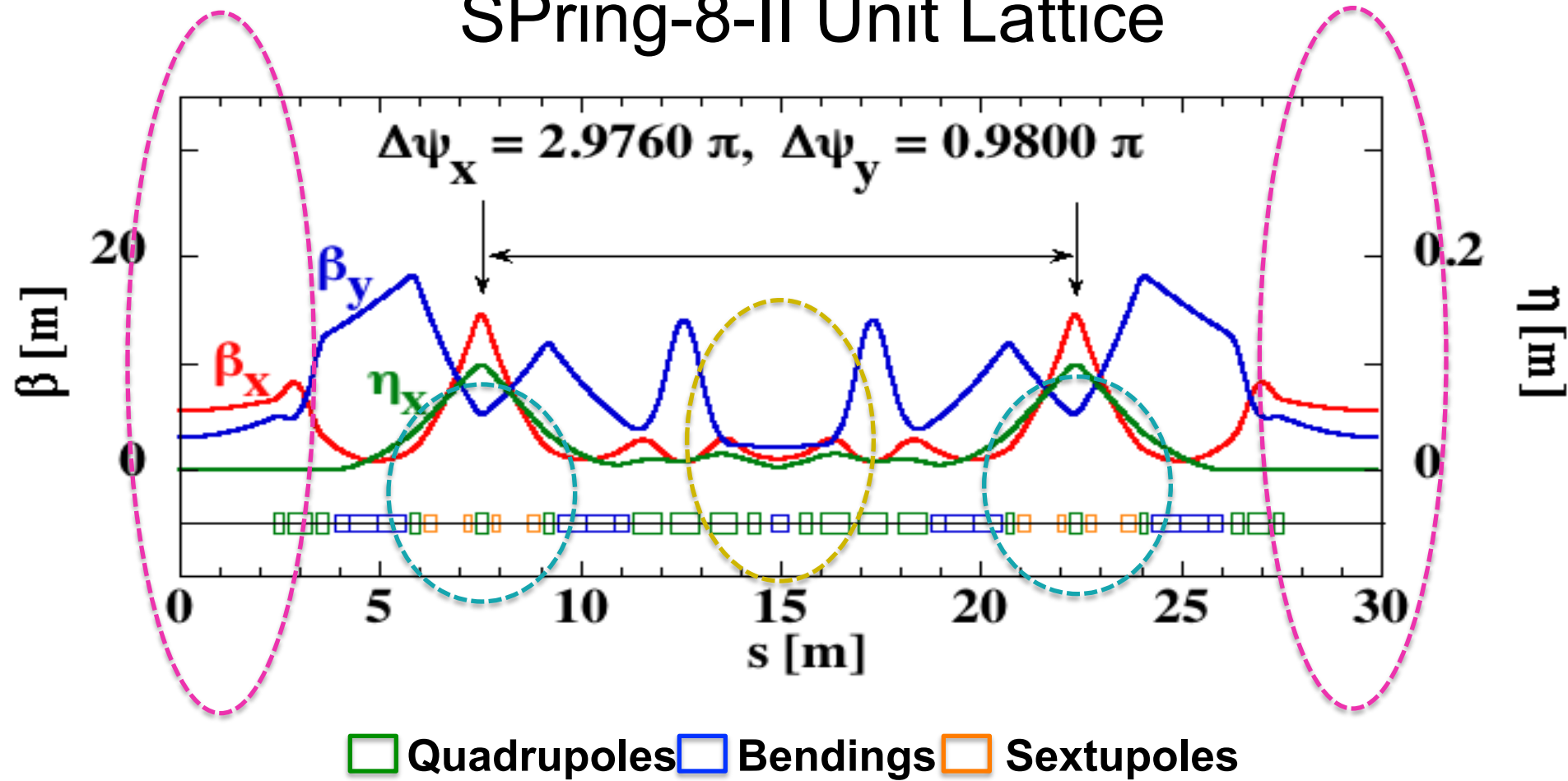
γ : Lorentz factor
 θ : Bending angle
 ρ : Bending radius
 H : H-function
 J_x : Damping partition number

Additional reduction schemes:

2. Reduction of stored energy (γ) with the help of advanced undulator design
3. Optimization of dipole field (ρ) in a dipole and / or inside unit cell)
4. Damping enhancement ($\langle H/\rho^3 \rangle / \langle 1/\rho^2 \rangle$) by additional radiation

4. Approach to coherent X-rays (4)

SPring-8-II Unit Lattice



4. Approach to coherent X-rays (5)

Main Parameter	New Optics	Present Optics
Energy (GeV)	6	8
Circumference (m)	1435.4	1435.9
Unit cell structure	5 BMs	2 BMs
Ring structure	2 Injection Cells + 42 Unit Cells + 4 Straight Cells	44 Unit Cells + 4 Straight Cells
ID straight length (m)	4.68	6.65
Natural emittance (nmrad)	0.15 (Achro, w/o und) ~0.10 (Achro, w und)	2.4 (NA) 6.7 (Achro)
Coupling ratio (%)	10	0.2
Stored current (mA)	100	100
Filling pattern	Multi-bunches	Multi-/Several bunches
Beam lifetime (hr)	<10	10~100

4. Approach to coherent X-rays (6)

Main Parameter

New Optics

Present Optics

ID straight		
β Function @ID (β_x, β_y) (m)	(5.5, 3.0)	(31.2, 5.0)
Dispersion η_x @ID (m)	0.0	0.146
Beam sizes @ID (σ_x, σ_y) (μm)	(24.0, 5.6)	(316, 4.9)
Angular div. @ID ($\sigma_{x'}, \sigma_{y'}$) (μrad)	(4.4, 1.9)	(8.8, 1.0)
Bending magnet BM1		
Critical photon energy (keV)	13.9	28.9
β Function (β_x, β_y) (m)	(1.8, 14)	(2.9, 28)
Dispersion η_x (m)	0.00016	0.039
Bending magnet BM2		
Critical photon energy (keV)	22.8	28.9
β Function (β_x, β_y) (m)	(0.9, 1.9)	(2.4, 31)
Dispersion η_x (m)	0.0016	0.059

4. Approach to coherent X-rays (7)

Spectrum of Standard Undulator Radiation

