



Coherence: from Undulators to Free Electron Lasers

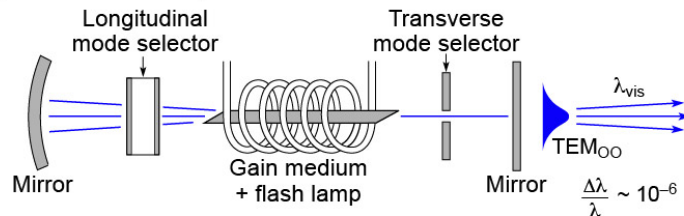
David Attwood
University of California, Berkeley

Cheiron School
September 25, 2014
SPring-8

Coherence



Laser Cavity



Spatial and Temporal Coherence

$$d \cdot 2\theta \big|_{\text{FWHM}} \approx \frac{\lambda}{2}$$

$$\ell_{\text{coh}} = \lambda^2 / 2\Delta\lambda$$

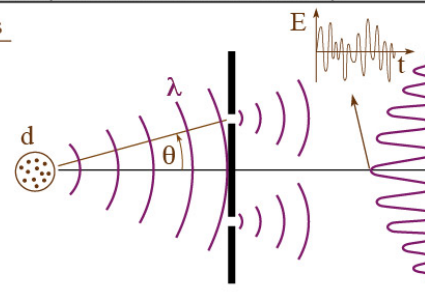
$$\tau_{\text{coh}} = \ell_{\text{coh}} / c$$

$$\Delta E \cdot \Delta \tau \big|_{\text{FWHM}} \geq 1.82 \text{ eV} \cdot \text{fsec}$$

Young's Double Slit (uncorrelated emitters)

N uncorrelated emitters

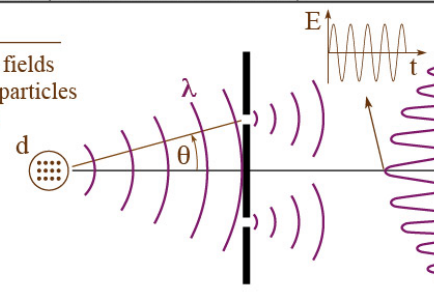
- Self-interference only
- Electric fields chaotic
- Intensities add
- Radiated power $\sim N$



Young's Double Slit (correlated emitters)

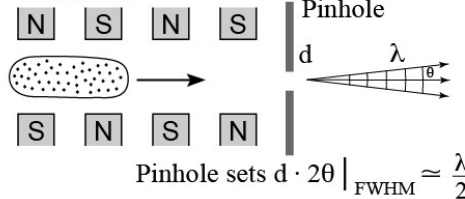
N correlated emitters

- Phase coherent electric fields
- Electric fields from all particles interfere constructively
- Radiated power $\sim N^2$

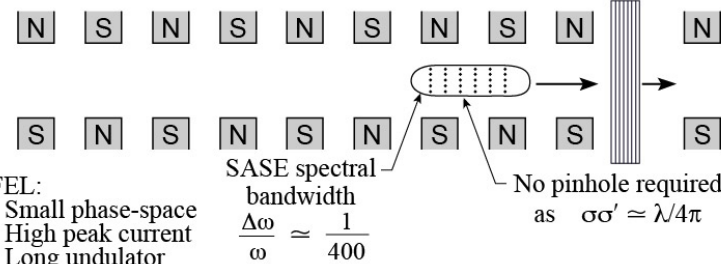


Undulators and Free-Electron Laser

Undulator



SASE FEL



- FEL:
- Small phase-space
 - High peak current
 - Long undulator

SASE spectral bandwidth
 $\frac{\Delta\omega}{\omega} \approx \frac{1}{400}$

No pinhole required
as $\sigma\sigma' \approx \lambda/4\pi$

After "self-seeding"
spectral bandwidth
 $\Delta\omega/\omega \approx 1/2000$
 $\ell_{\text{coh}} = \lambda^2 / 2\Delta\lambda \sim 10^3 \lambda$

$d \cdot 2\theta \big|_{\text{FWHM}} \approx \lambda/2$

Spatially, temporally coherent, but jumbled phase

Coherence_Collage_Sept2014.ai

Coherence



co·here \kō-'hi(ə)r\ *vi* [L *cohaerēre*, fr. *co-* + *haerēre* to stick]
1 a : to hold together firmly as parts of the same mass **b** : ADHERE
c : to display cohesion **2** : to consist of parts that cohere **3 a** : to become united in principles, relationships, or interests **b** : to be logically or aesthetically consistent **syn** see STICK
co·her·ence \kō-'hīr-ən(t)s, -'her-\ *n* : the quality or state of cohering; *esp* : systematic connection esp. in logical discourse
co·her·en·cy \-ən-sē\ *n* : COHERENCE
co·her·ent \kō-'hīr-ənt, -'her-\ *adj* [MF or L; MF *cohérent*, fr. L *cohaerent-*, *cohaerens*, prp. of *cohaerēre*] **1** : having the quality of cohering **2** : logically consistent **3** : having waves in phase and of one wavelength (<~ light> — **co·her·ent·ly** *adv*

Webster's 7th Collegiate Dictionary
(1971)

co·her·ence (kō-hīr'əns, -hēr'-) also **co·her·en·cy** (-ən-sē)
n. The quality or state of cohering, esp. logical or orderly relationship of parts.
co·her·ent (kō-hīr'ənt, -hēr'-) *adj*. **1**. Sticking together; cohering. **2**. Marked by an orderly or logical relation of parts that affords comprehension or recognition: coherent speech. **3**. *Physics*. Of or pertaining to waves with a continuous relationship among phases. **4**. Of or pertaining to a system of units of measurement in which a small number of basic

American Heritage 2nd College Edition
Dictionary (1985)

- Coherence (physics), an ideal property of waves that enables stationary (i.e. temporally and spatially constant) interference
- Coherence time, the time over which a propagating wave (especially a laser or maser beam) may be considered coherent; the time interval within which its phase is, on average, predictable

Wikipedia,
the free encyclopedia



Born and Wolf, Chapter 10

Interference and diffraction with partially coherent light

10.1 Introduction

So far we have been mainly concerned with monochromatic light produced by a point source. Light from a real physical source is never strictly monochromatic, since even the sharpest spectral line has a finite width. Moreover, a physical source is not a point source, but has a finite extension, consisting of very many elementary radiators (atoms). The disturbance produced by such a source may be expressed, according to Fourier's theorem, as the sum of strictly monochromatic and therefore infinitely long wave trains. The elementary monochromatic theory is essentially concerned with a single component of this Fourier representation.

Physical sources are of finite size and duration

In a constant field produced by the rapid oscillations of the electric and magnetic fields, the time interval between two successive peaks of the electric field is of the order of 10^{-15} s.

Consider next the light disturbances at two points P_1 and P_2 in a wave field produced by an extended quasi-monochromatic source. For simplicity assume that the wave field is in vacuum and that P_1 and P_2 are many wavelengths away from the source. We may expect that, when P_1 and P_2 are close enough to each other, the fluctuations of the amplitudes at these points, and also the fluctuations of the phases, will not be independent. It is reasonable to suppose that, if P_1 and P_2 are so close to each other that the difference $\Delta S = SP_1 - SP_2$ between the paths from each source point S is small compared to the mean wavelength $\bar{\lambda}$, then the fluctuations at P_1 and P_2 will effectively be the same; and that some correlation between the fluctuations will exist even for greater separations of P_1 and P_2 , provided that for all source points the path difference ΔS does not exceed the coherence length $c\Delta t \sim c/\Delta\nu = \bar{\lambda}^2/\Delta\lambda$. We are thus led to the concept of a region of coherence around any point P in a wave field.

In order to describe adequately a wave field produced by a finite polychromatic source it is evidently desirable to introduce some measure for the correlation that exists between the vibrations at different points P_1 and P_2 in the field. We must expect such a measure to be closely related to the sharpness of the interference fringes which would result on combining the vibrations from the two points. We must expect sharp fringes when the correlation is high (e.g. when the light at P_1 and P_2 comes from a very small source of a narrow spectral range), and no fringes at all in the absence of correlation (e.g. when P_1 and P_2 each receive light from a different physical source). We described these situations by the terms 'coherent' and 'incoherent' respectively. In general neither of these situations is realized and we may speak of vibrations which are partially coherent.

Regions of coherence

$$\ell_{\text{coh}} = c\Delta t = \lambda^2/2\Delta\lambda$$

Normalized degree of coherence

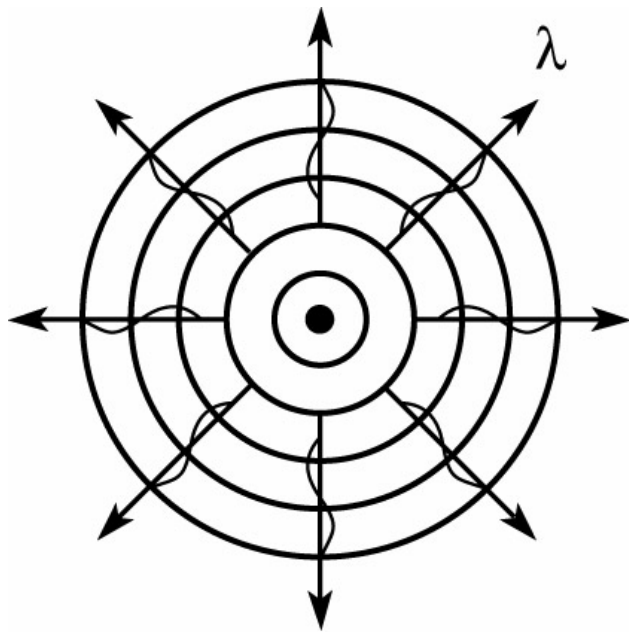
$$\gamma_{12}(\tau) \equiv \frac{\langle E_1(t + \tau)E_2^*(t) \rangle}{\sqrt{\langle |E_1|^2 \rangle} \sqrt{\langle |E_2|^2 \rangle}}$$

Coherent

Incoherent

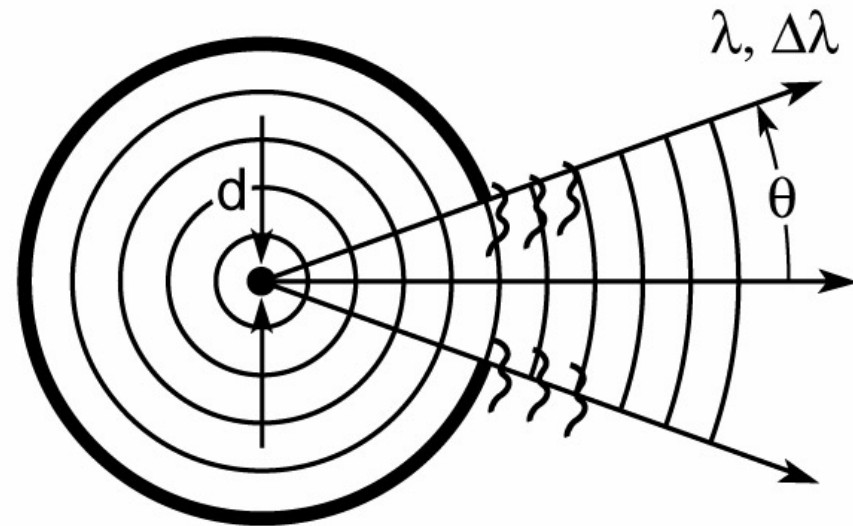
Partially incoherent

Coherence, partial coherence, and incoherence



Point source oscillator

$$-\infty < t < \infty$$



Source of finite size,
divergence, and duration

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Spatial and temporal coherence



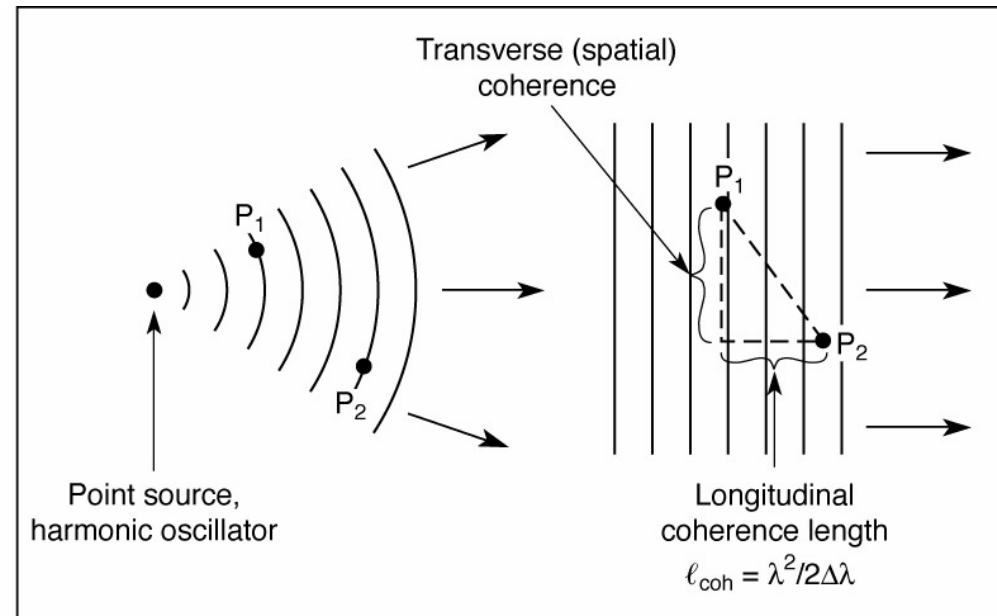
Mutual coherence factor

$$\Gamma_{12}(\tau) \equiv \langle E_1(t + \tau) E_2^*(t) \rangle \quad (8.1)$$

Normalize degree of spatial coherence
(complex coherence factor)

$$\mu_{12} = \frac{\langle E_1(t) E_2^*(t) \rangle}{\sqrt{\langle |E_1|^2 \rangle} \sqrt{\langle |E_2|^2 \rangle}} \quad (8.12)$$

A high degree of coherence ($\mu \rightarrow 1$) implies an ability to form a high contrast interference (fringe) pattern. A low degree of coherence ($\mu \rightarrow 0$) implies an absence of interference, except with great care. In general radiation is partially coherent.



Longitudinal (temporal) coherence length

$$\ell_{\text{coh}} = \frac{\lambda^2}{2 \Delta \lambda} \quad (8.3)$$

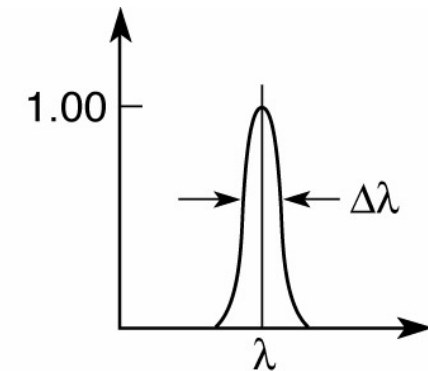
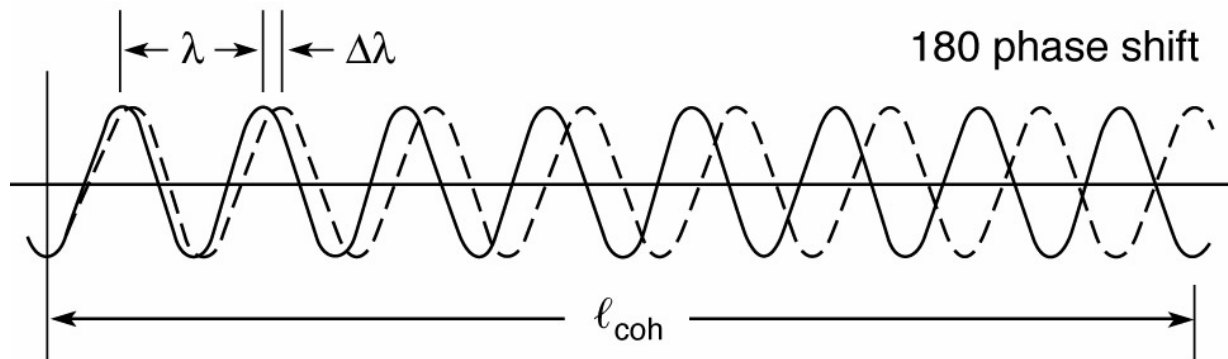
Full spatial (transverse) coherence

$$d \cdot \theta = \lambda / 2\pi \quad (8.5)$$

Marching band and coherence lengths



Spectral bandwidth and longitudinal coherence length



Define a coherence length ℓ_{coh} as the distance of propagation over which radiation of spectral width $\Delta\lambda$ becomes 180° out of phase. For a wavelength λ propagating through N cycles

$$\ell_{\text{coh}} = N\lambda$$

and for a wavelength $\lambda + \Delta\lambda$, a half cycle less $(N - \frac{1}{2})$

$$\ell_{\text{coh}} = (N - \frac{1}{2}) (\lambda + \Delta\lambda)$$

Equating the two

$$N = \lambda / 2\Delta\lambda$$

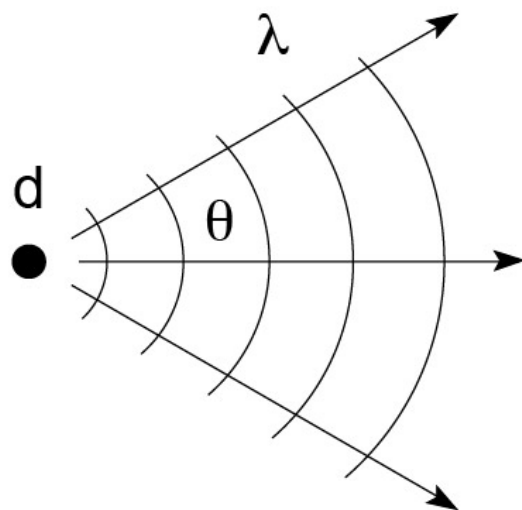
so that

$$\boxed{\ell_{\text{coh}} = \frac{\lambda^2}{2 \Delta\lambda}} \quad (8.3)$$



A practical interpretation of spatial coherence

- Associate spatial coherence with a spherical wavefront.
- A spherical wavefront implies a point source.
- How small is a “point source”?



From Heisenberg's Uncertainty Principle ($\Delta x \cdot \Delta p \geq \frac{\hbar}{2}$), the smallest source size “d” you can resolve, with wavelength λ and half angle θ , is

$$d \cdot \theta = \frac{\lambda}{2\pi} \quad (\text{rms})$$

Partially coherent radiation approaches uncertainty principle limits



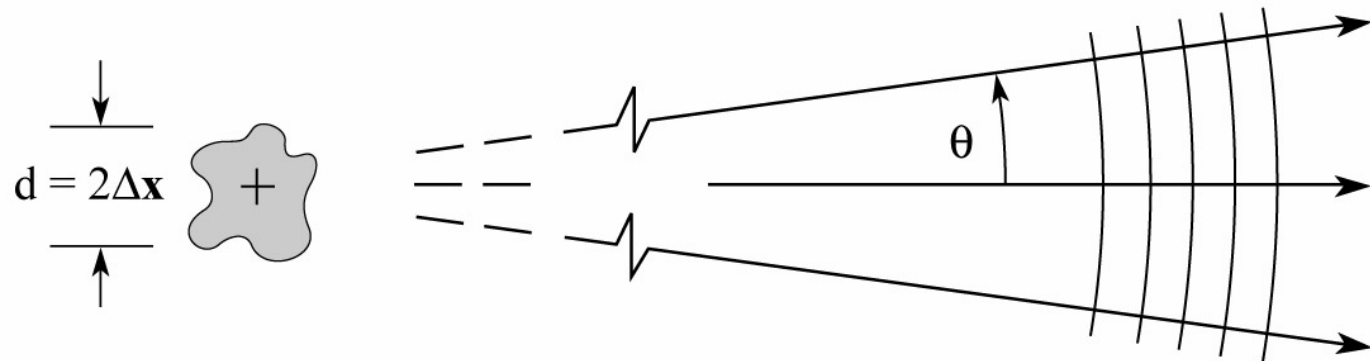
$$\Delta \mathbf{x} \cdot \Delta \mathbf{p} \geq \hbar/2 \quad (8.4)$$

Standard deviations of Gaussian distributed functions
(Tipler, 1978, pp. 174-189)

$$\Delta \mathbf{x} \cdot \hbar \Delta \mathbf{k} \geq \hbar/2$$

$$\Delta \mathbf{x} \cdot \mathbf{k} \Delta \theta \geq 1/2$$

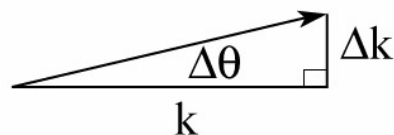
$$2\Delta \mathbf{x} \cdot \Delta \theta \geq \lambda/2\pi$$



Note:

$$\Delta \mathbf{p} = \hbar \Delta \mathbf{k}$$

$$\Delta \mathbf{k} = \mathbf{k} \Delta \theta$$



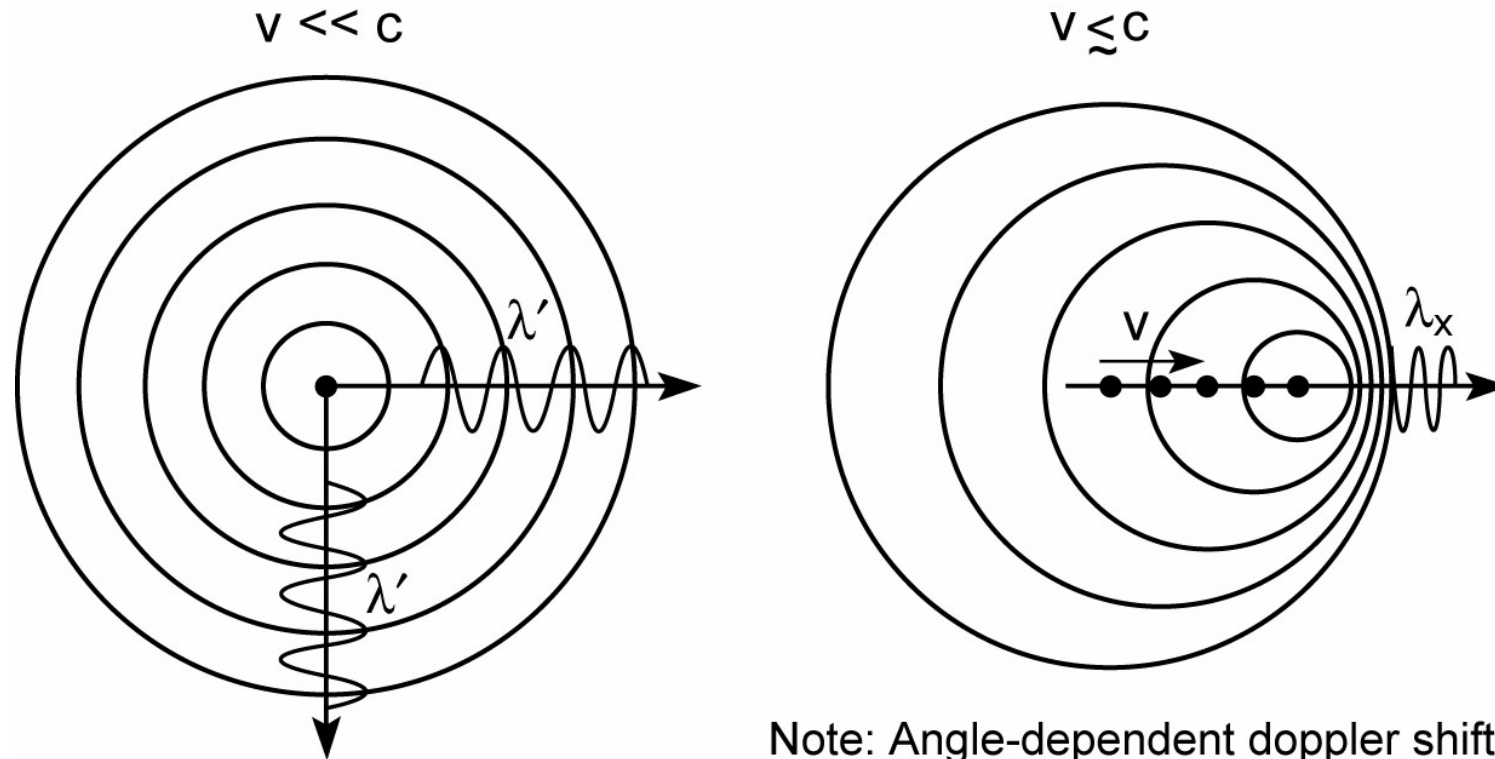
Spherical wavefronts occur
in the limiting case

$$\left. \begin{array}{l} d \cdot \theta = \lambda/2\pi \\ \text{(spatially coherent)} \end{array} \right\} \frac{1}{\sqrt{e}} \text{ quantities}$$

or

$$(d \cdot 2\theta)_{\text{FWHM}} \approx \lambda/2 \left. \vphantom{\frac{1}{\sqrt{e}}} \right\} \text{FWHM quantities}$$

X-rays from relativistic electrons



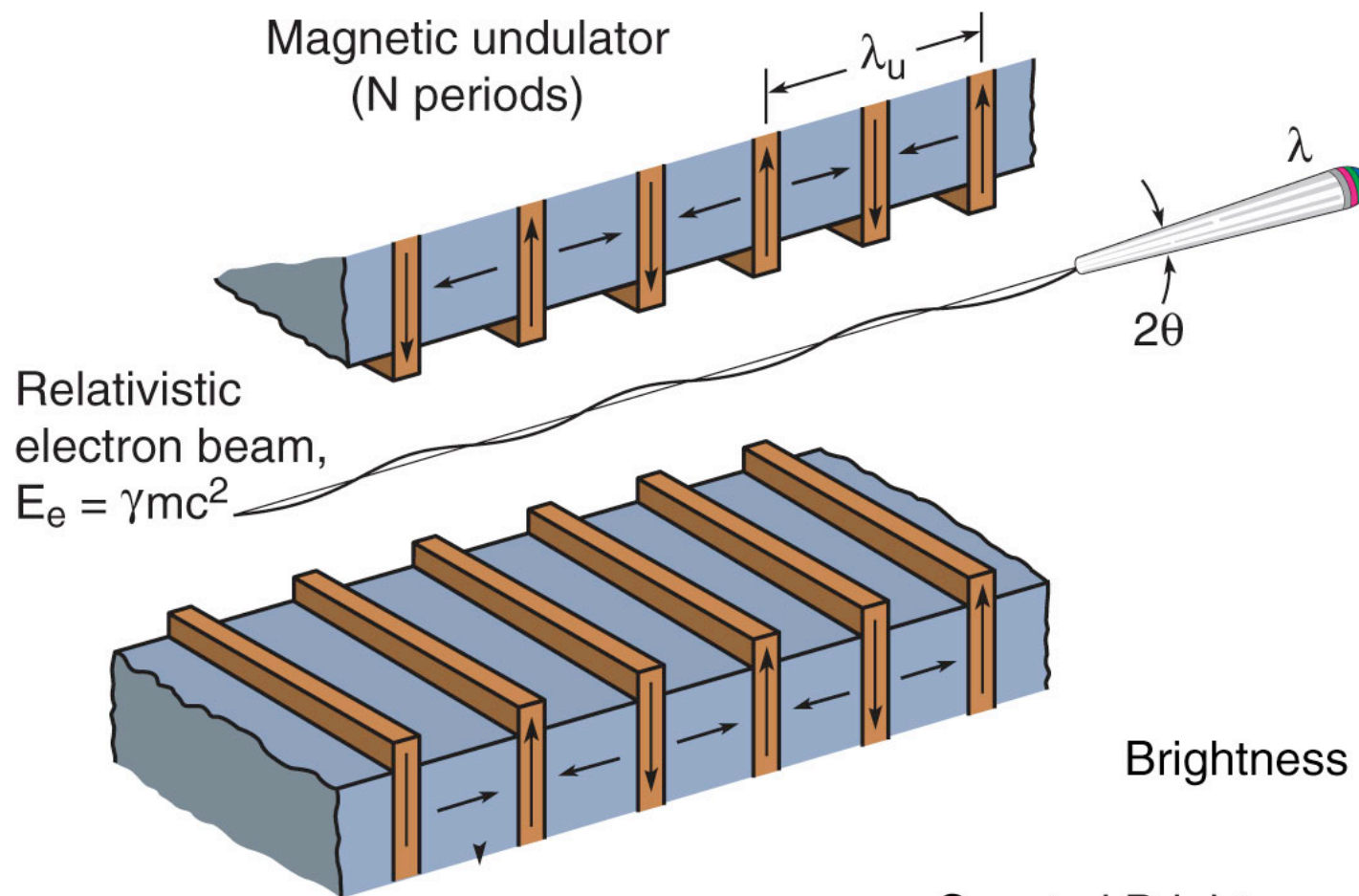
Note: Angle-dependent doppler shift

$$\lambda = \lambda' \left(1 - \frac{v}{c} \cos\theta\right) \quad \lambda = \lambda' \gamma \left(1 - \frac{v}{c} \cos\theta\right)$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Courtesy of John Madey

Undulator radiation from a small electron beam radiating into a narrow forward cone is very bright



$$\lambda \approx \frac{\lambda_u}{2\gamma^2}$$

$$\theta_{\text{cen}} \approx \frac{1}{\gamma\sqrt{N}}$$

$$\left[\frac{\Delta\lambda}{\lambda} \right]_{\text{cen}} = \frac{1}{N}$$

$$\text{Brightness} = \frac{\text{photon flux}}{(\Delta A) (\Delta\Omega)}$$

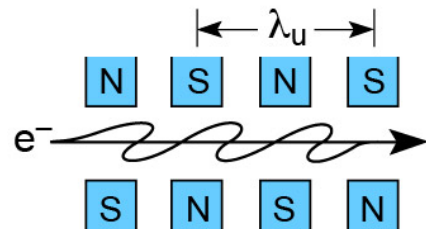
$$\text{Spectral Brightness} = \frac{\text{photon flux}}{(\Delta A) (\Delta\Omega) (\Delta\lambda/\lambda)}$$

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Undulator radiation

Laboratory Frame of Reference

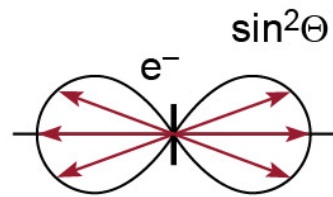


$$E = \gamma mc^2$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$N = \# \text{ periods}$

Frame of Moving e^-



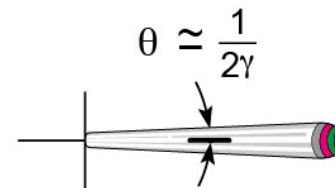
e^- radiates at the Lorentz contracted wavelength:

$$\lambda' = \frac{\lambda_u}{\gamma}$$

Bandwidth:

$$\frac{\lambda'}{\Delta \lambda'} \simeq N$$

Frame of Observer



Doppler shortened wavelength on axis:

$$\lambda = \lambda' \gamma (1 - \beta \cos \theta)$$

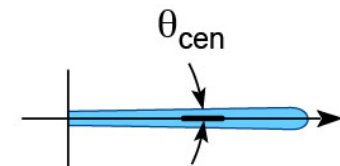
$$\lambda = \frac{\lambda_u}{2\gamma^2} (1 + \gamma^2 \theta^2)$$

Accounting for transverse motion due to the periodic magnetic field:

$$\lambda = \frac{\lambda_u}{2\gamma^2} \left(1 + \frac{K^2}{2} + \gamma^2 \theta^2 \right)$$

where $K = eB_0\lambda_u / 2\pi mc$

Following Monochromator



$$\text{For } \frac{\Delta \lambda}{\lambda} \simeq \frac{1}{N}$$

$$\theta_{\text{cen}} \simeq \frac{1}{\gamma \sqrt{N}}$$

typically

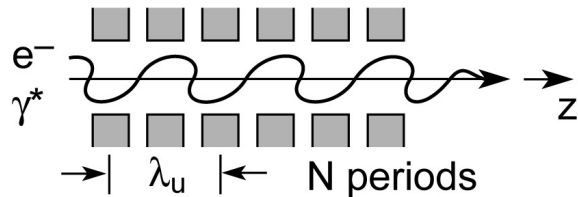
$$\theta_{\text{cen}} \simeq 10\text{-}30 \text{ } \mu\text{rad}$$

Following Albert Hoffman

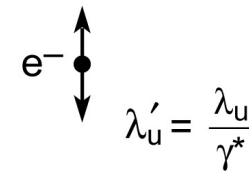
Calculating Power in the Central Radiation Cone: Using the well known “dipole radiation” formula by transforming to the frame of reference moving with the electrons



x, z, t laboratory frame of reference



x', z', t' frame of reference moving with the average velocity of the electron



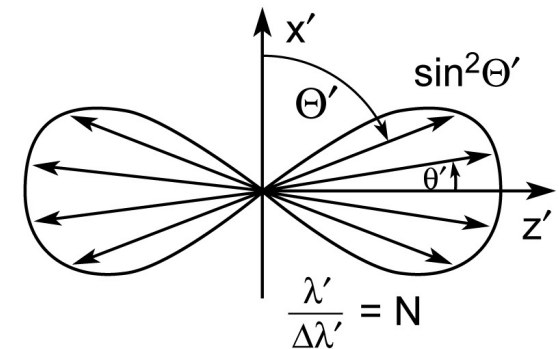
Lorentz transformation

Determine x, z, t motion:

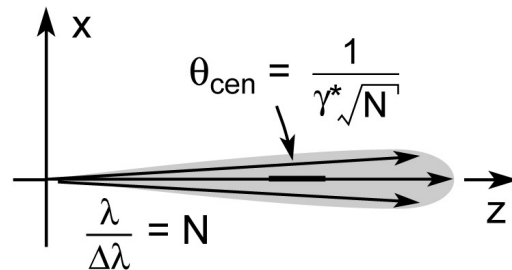
$$\frac{d\mathbf{p}}{dt} = -e (\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

Dipole radiation:

$$\frac{dP'}{d\Omega'} = \frac{e^2 a'^2 \sin^2 \Theta'}{16\pi^2 \epsilon_0 c^3}$$



Lorentz transformation



$$\bar{P}_{\text{cen}} = \frac{\pi e \gamma^2 I}{\epsilon_0 \lambda_u} \frac{K^2}{(1 + K^2/2)^2}$$

$$\frac{dP'}{d\Omega'} = \frac{e^2 c \gamma^2}{4\epsilon_0 \lambda_u^2} \frac{K^2}{(1 + K^2/2)^2} (1 - \sin^2 \theta' \cos^2 \phi') \cos^2 \omega'_u t'$$

Power in the central cone

$$\lambda_x = \frac{\lambda_u}{2\gamma^2} \left(1 + \frac{K^2}{2} + \gamma^2 \theta^2\right)$$

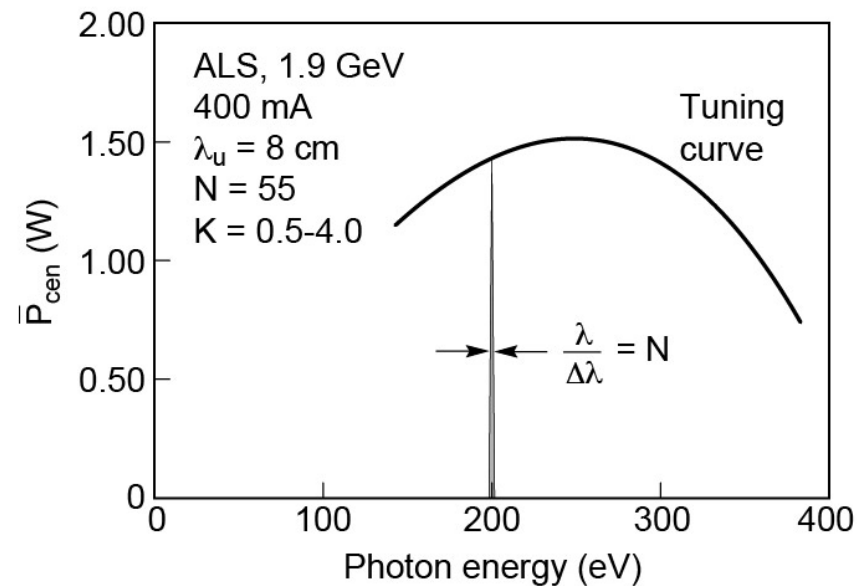
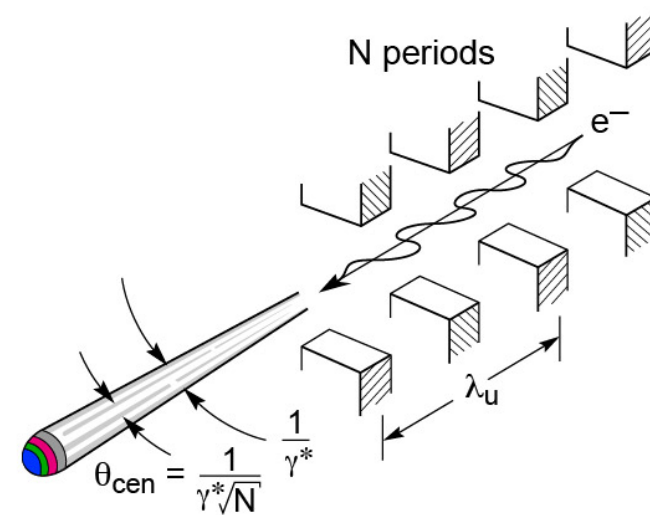
$$\bar{P}_{\text{cen}} = \frac{\pi e \gamma^2 I}{\epsilon_0 \lambda_u} \frac{K^2}{\left(1 + \frac{K^2}{2}\right)^2} [JJ]^2$$

$$\theta_{\text{cen}} = \frac{1}{\gamma^* \sqrt{N}}$$

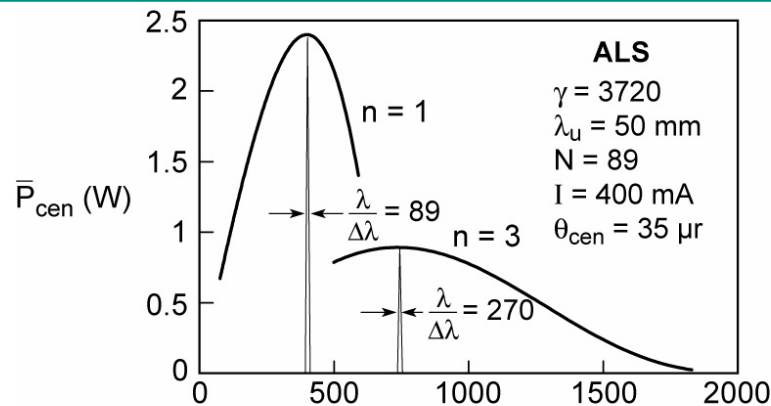
$$\left(\frac{\Delta\lambda}{\lambda}\right)_{\text{cen}} = \frac{1}{N}$$

$$K = \frac{eB_0 \lambda_u}{2\pi m_0 c}$$

$$\gamma^* = \gamma \sqrt{1 + \frac{K^2}{2}}$$



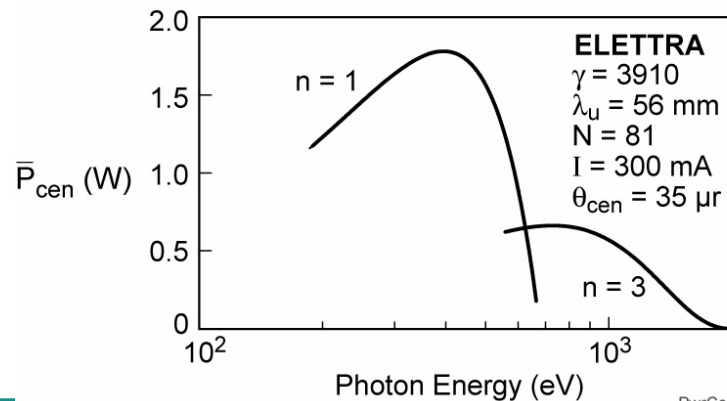
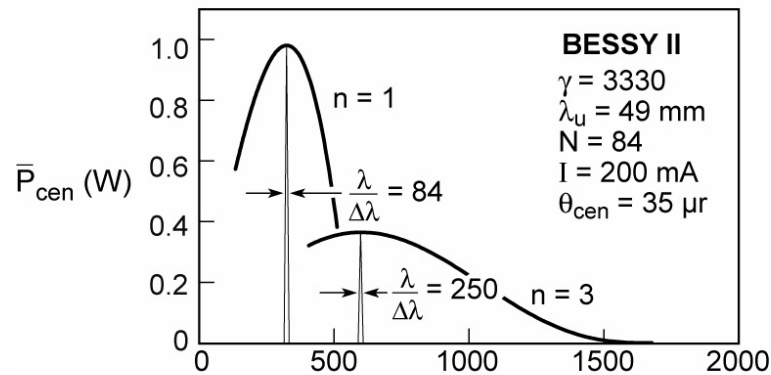
Power in the central radiation cone for three soft x-ray undulators



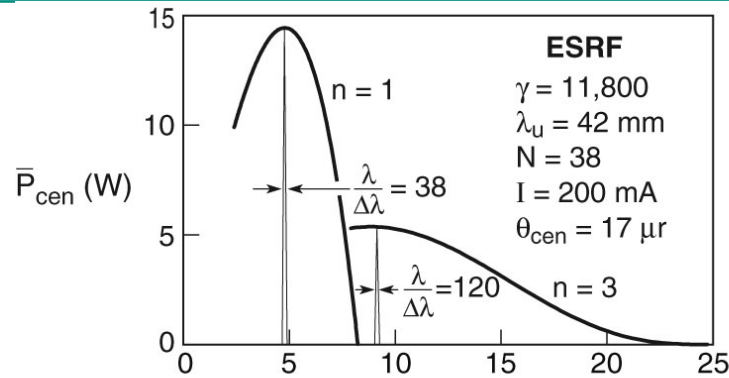
$$\theta_{\text{cen}} = \frac{1}{\gamma^* \sqrt{N}}$$

$$\left[\frac{\Delta\lambda}{\lambda} \right]_1 = \frac{1}{N}$$

$$\left[\frac{\Delta\lambda}{\lambda} \right]_3 = \frac{1}{3N}$$



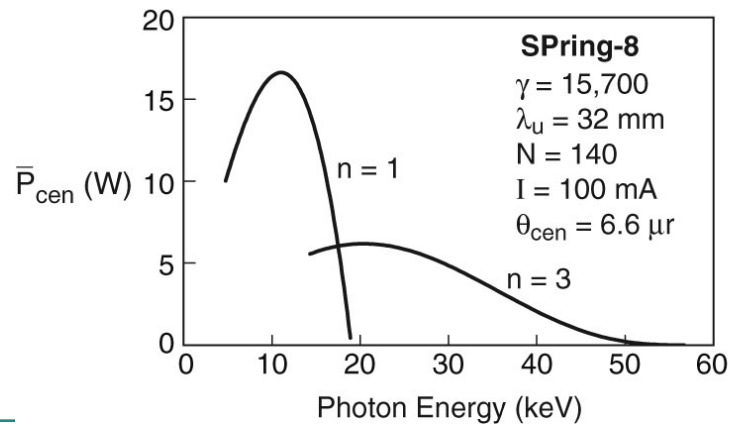
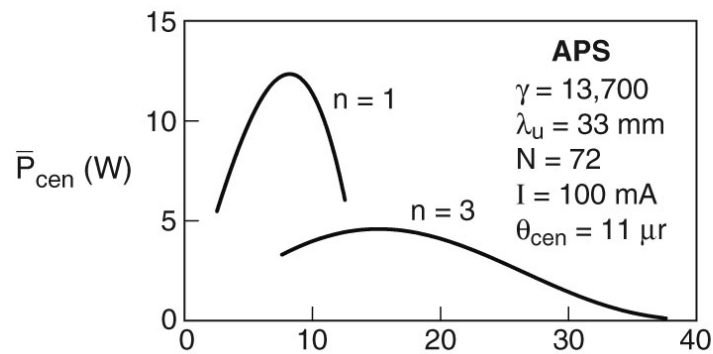
Power in the central radiation cone for three hard x-ray undulators



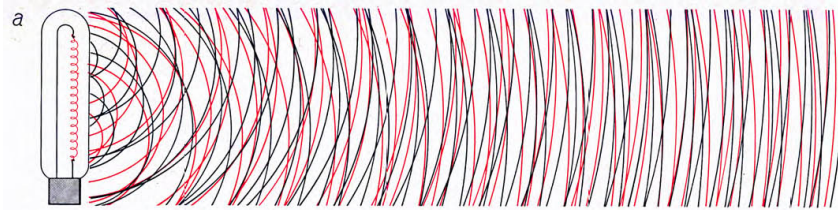
$$\theta_{\text{cen}} = \frac{1}{\gamma^* \sqrt{N}}$$

$$\left[\frac{\Delta\lambda}{\lambda} \right]_1 = \frac{1}{N}$$

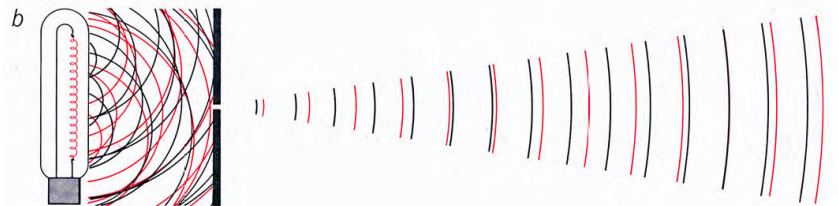
$$\left[\frac{\Delta\lambda}{\lambda} \right]_3 = \frac{1}{3N}$$



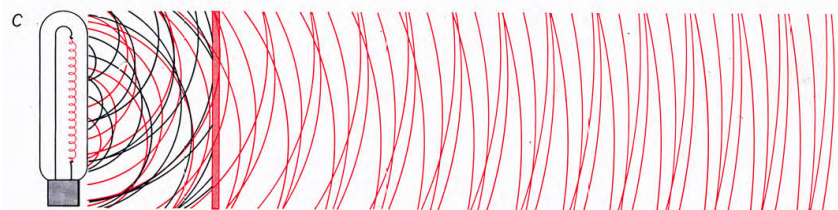
Ordinary light and laser light



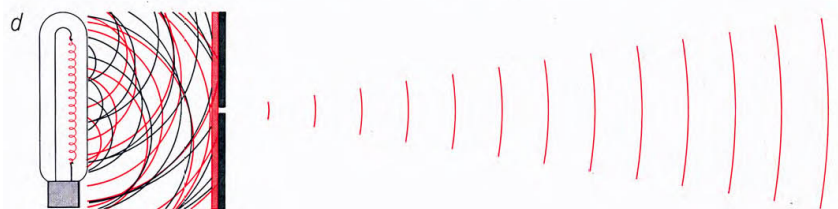
Ordinary thermal light source, atoms radiate independently.



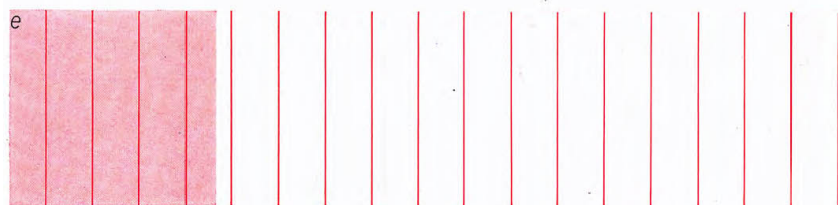
A pinhole can be used to obtain spatially coherent light, but at a great loss of power.



A color filter (or monochromator) can be used to obtain temporally coherent light, also at a great loss of power.



Pinhole and spectral filtering can be used to obtain light which is both spatially and temporally coherent but the power will be very small (tiny).



All of the laser light is both spatially and temporally coherent*.

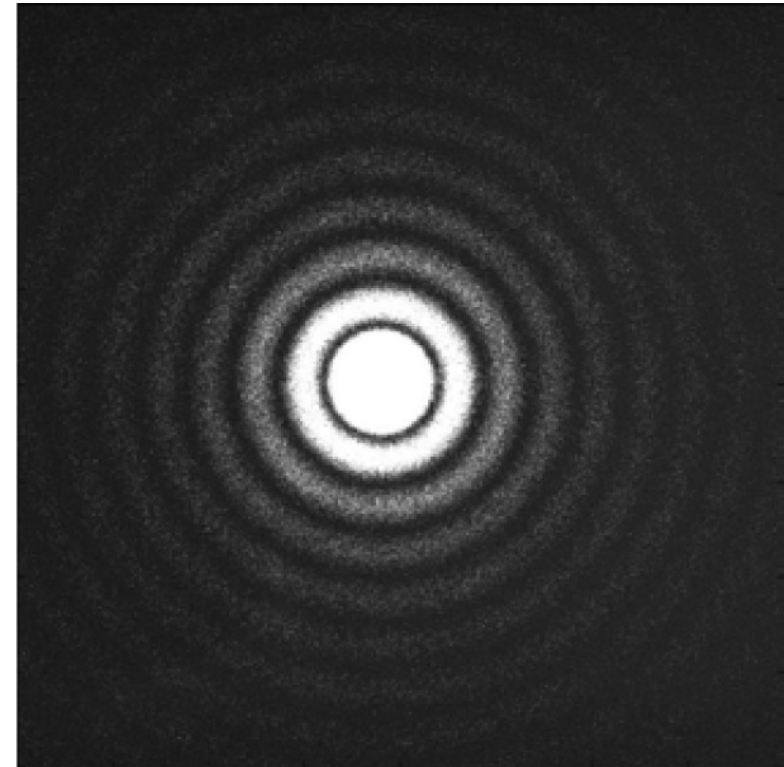
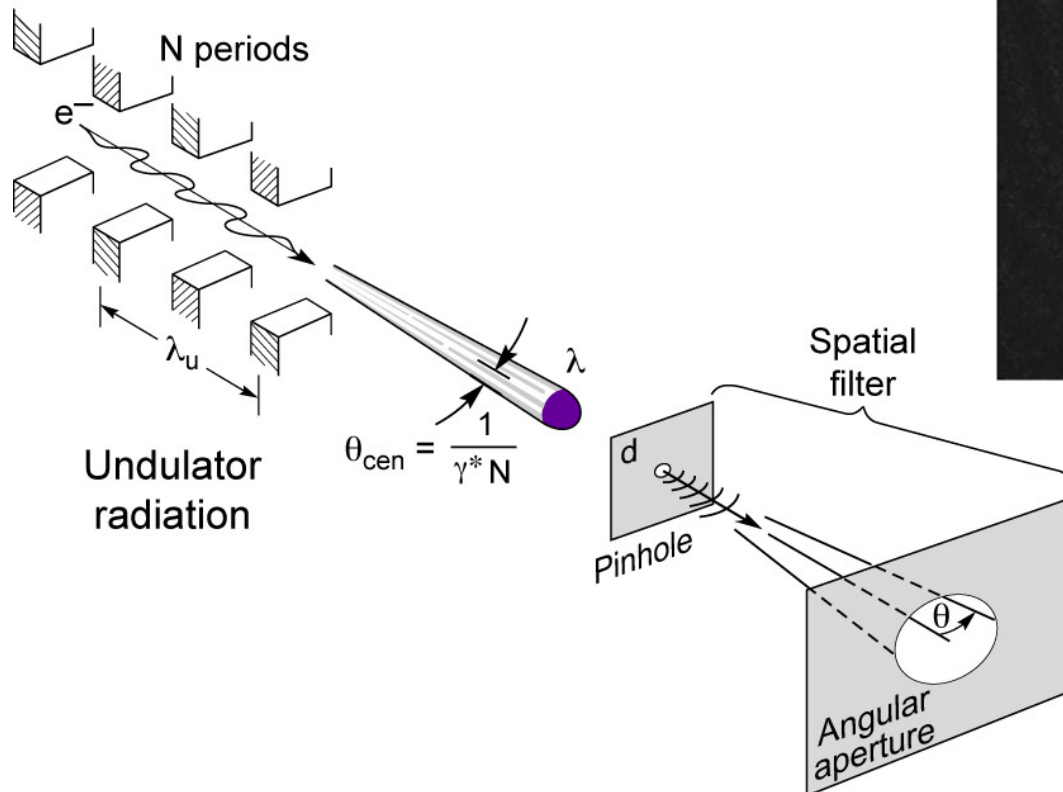
Arthur Schawlow, "Laser Light", Sci. Amer. 219, 120 (Sept. 1968)

Spatially coherent undulator radiation

$$l_{\text{coh}} = \lambda^2 / 2\Delta\lambda \quad \{\text{temporal (longitudinal) coherence}\} \quad (8.3)$$

$$d \cdot \theta = \lambda / 2\pi \quad \{\text{spatial (transverse) coherence}\} \quad (8.5)$$

$$\text{or } d \cdot 2\theta|_{\text{FWHM}} = 0.44 \lambda \quad (8.5^*)$$



$\lambda = 2.48 \text{ nm}$ (600 eV)

$d = 2.5 \mu\text{m}$

$t = 200 \text{ msec}$

ALS beamline 12.0.2

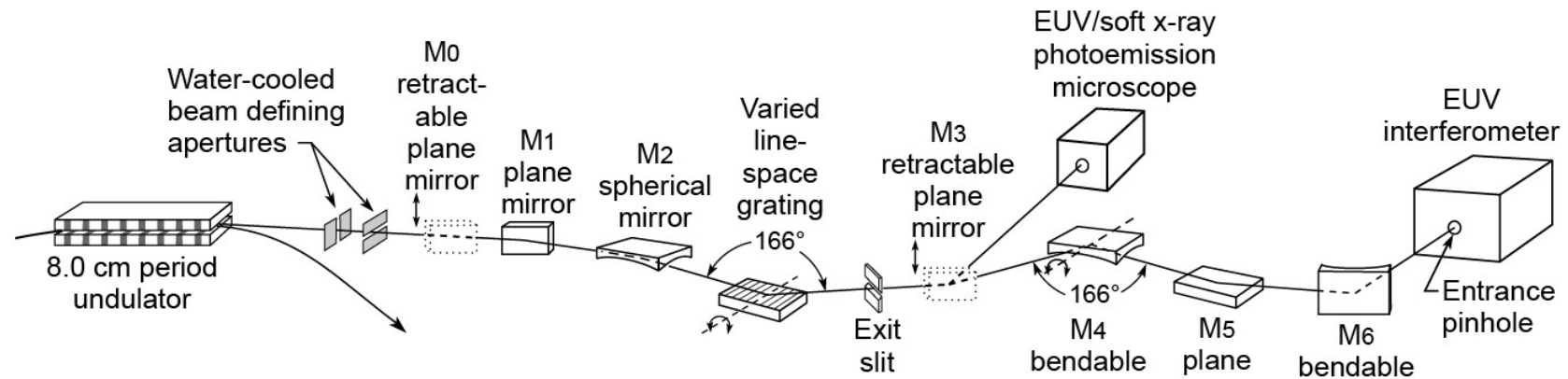
$\lambda_u = 80 \text{ mm}$, $N = 55$, $n = 3$

25 mm wide CCD at 410 mm

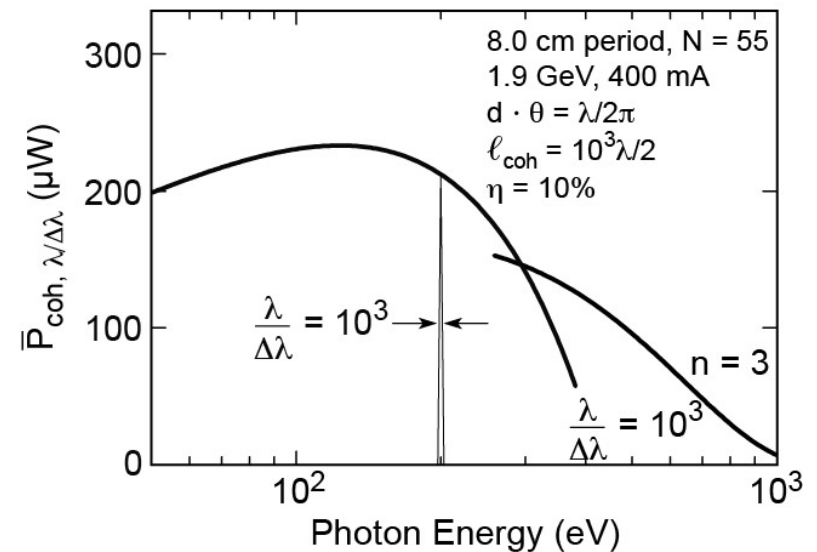
Courtesy of Kris Rosfjord, UCB

Spatially and spectrally filtered undulator radiation

- Pinhole filtering for full spatial coherence
- Monochromator for spectral filtering to $\lambda/\Delta\lambda > N$

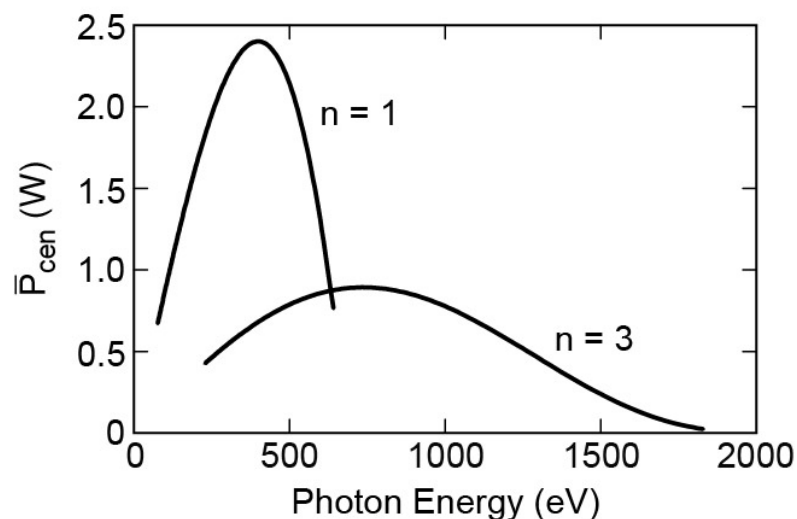


$$\bar{P}_{\text{coh}, \lambda/\Delta\lambda} = \underbrace{\eta}_{\text{beamline efficiency}} \underbrace{\frac{(\lambda/2\pi)^2}{(d_x \theta_x)(d_y \theta_y)}}_{\text{spatial filtering}} \cdot \underbrace{N \frac{\Delta\lambda}{\lambda}}_{\text{spectral filtering}} \cdot \bar{P}_{\text{cen}} \quad (8.10a)$$



Coherent power at the ALS

U5



1.9 GeV, 400 mA

$\lambda_u = 50$ mm, $N = 89$

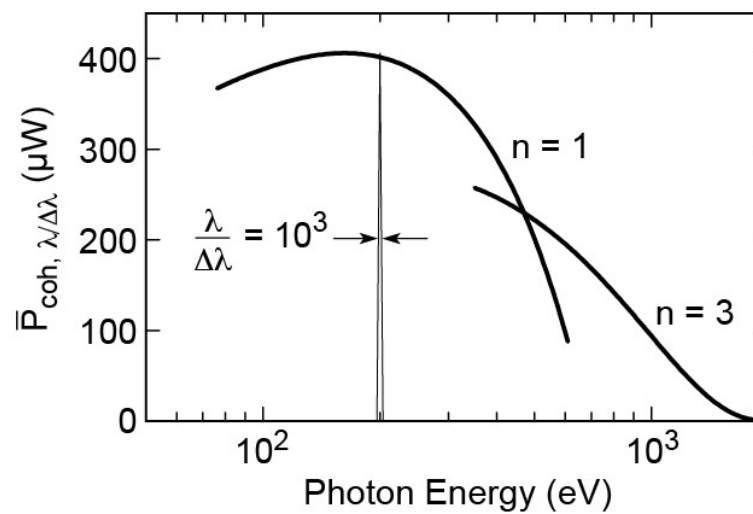
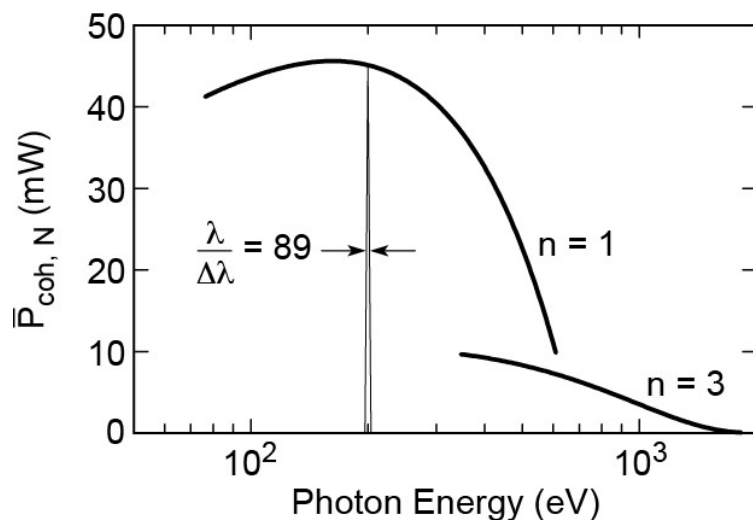
$0.5 \leq K \leq 4.0$

$\sigma_x = 260$ μm , $\sigma_x' = 23$ μr

$\sigma_y = 16$ μm , $\sigma_y' = 3.9$ μr

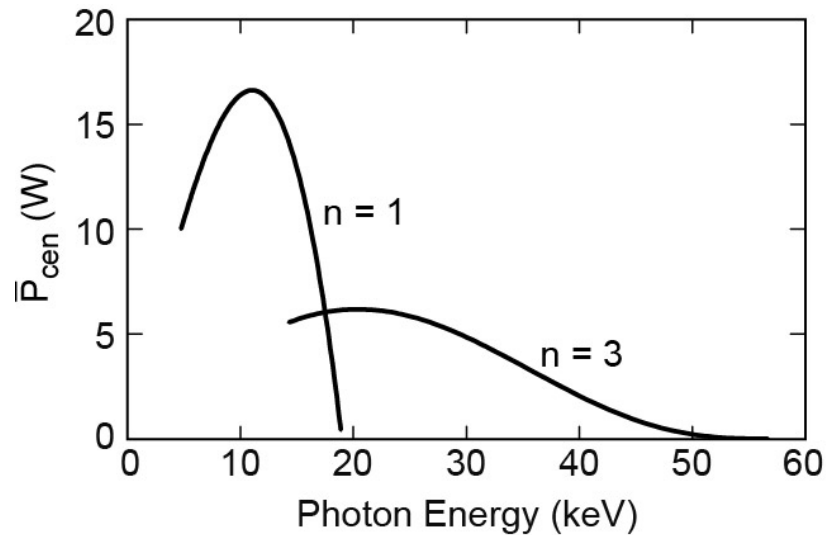
$\theta_{\text{cen}} = 35$ μrad

$\eta = 10\%$

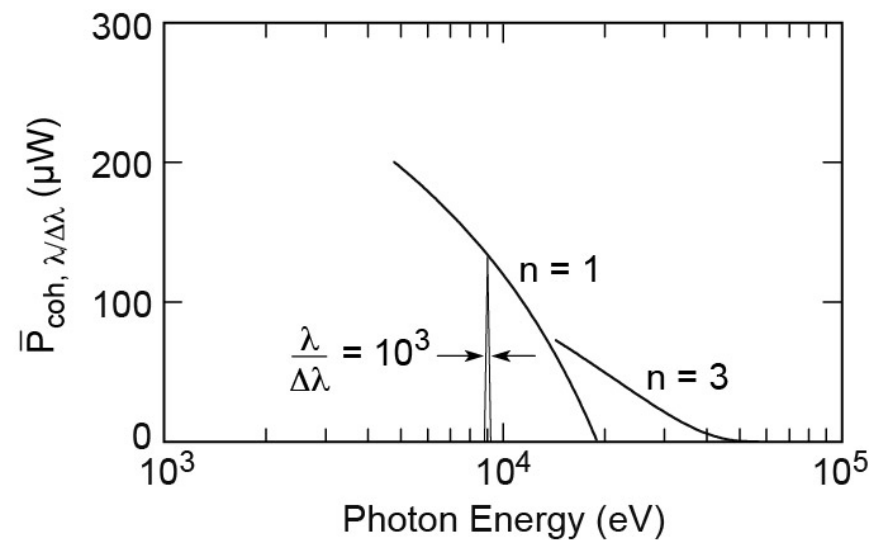
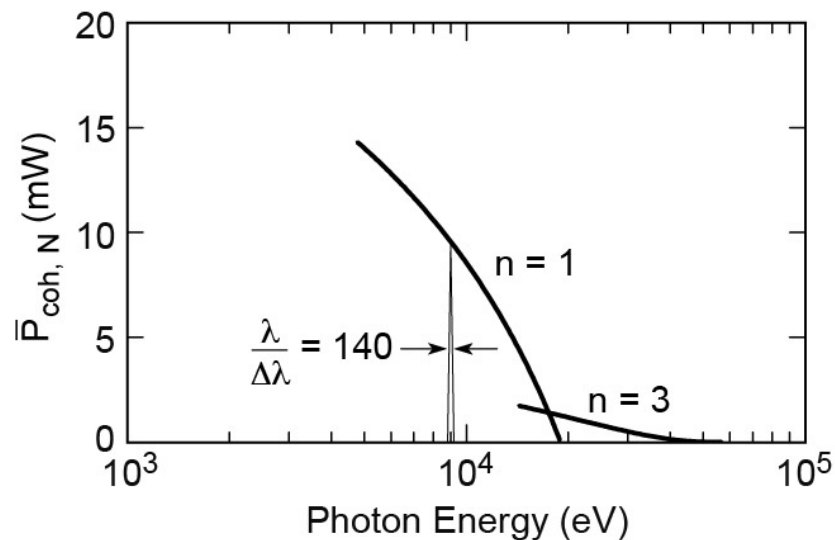




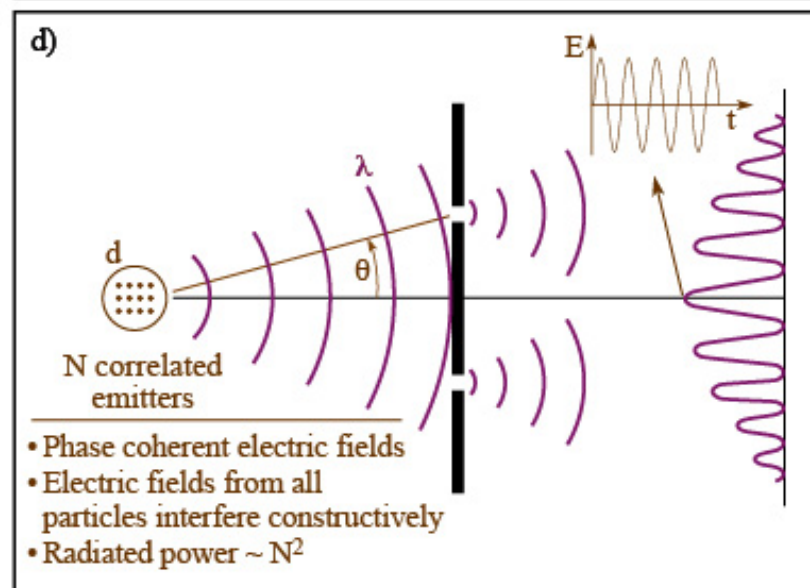
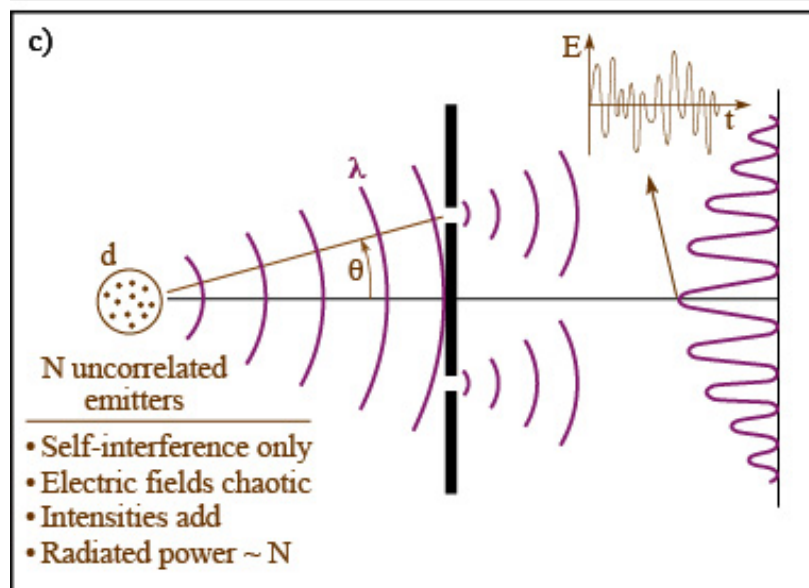
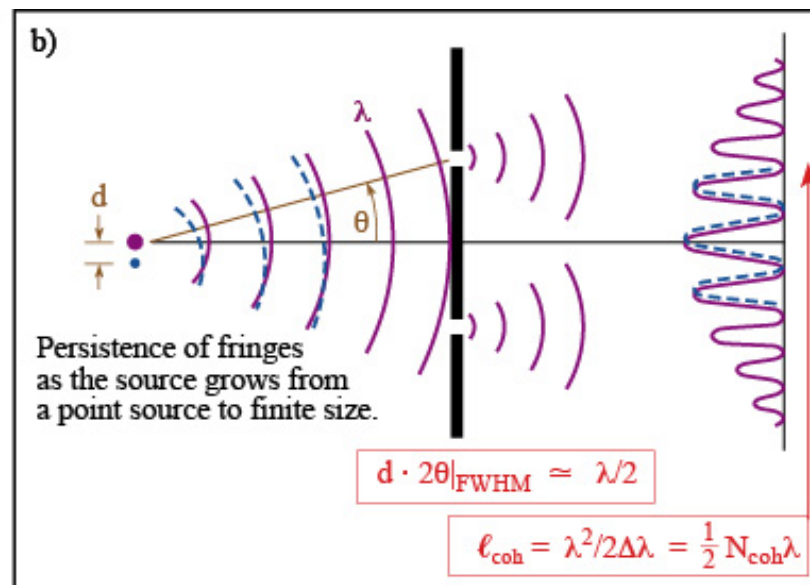
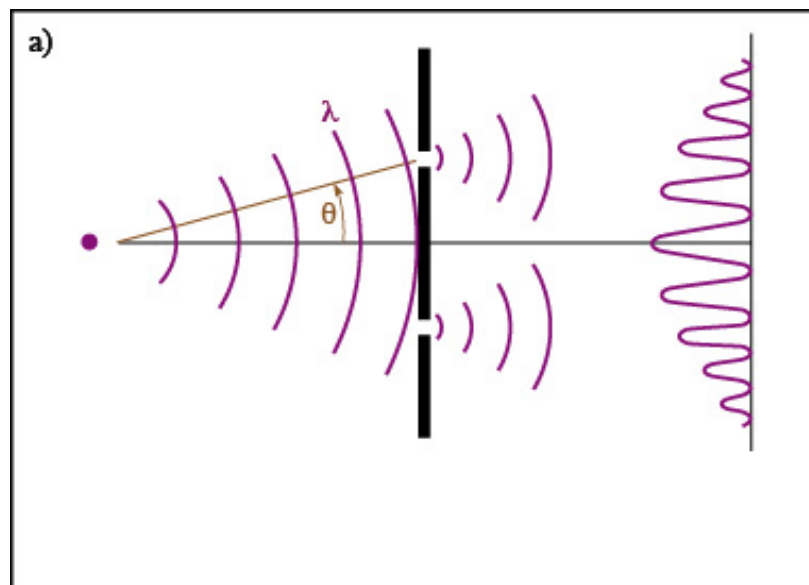
Coherent power at SPring-8



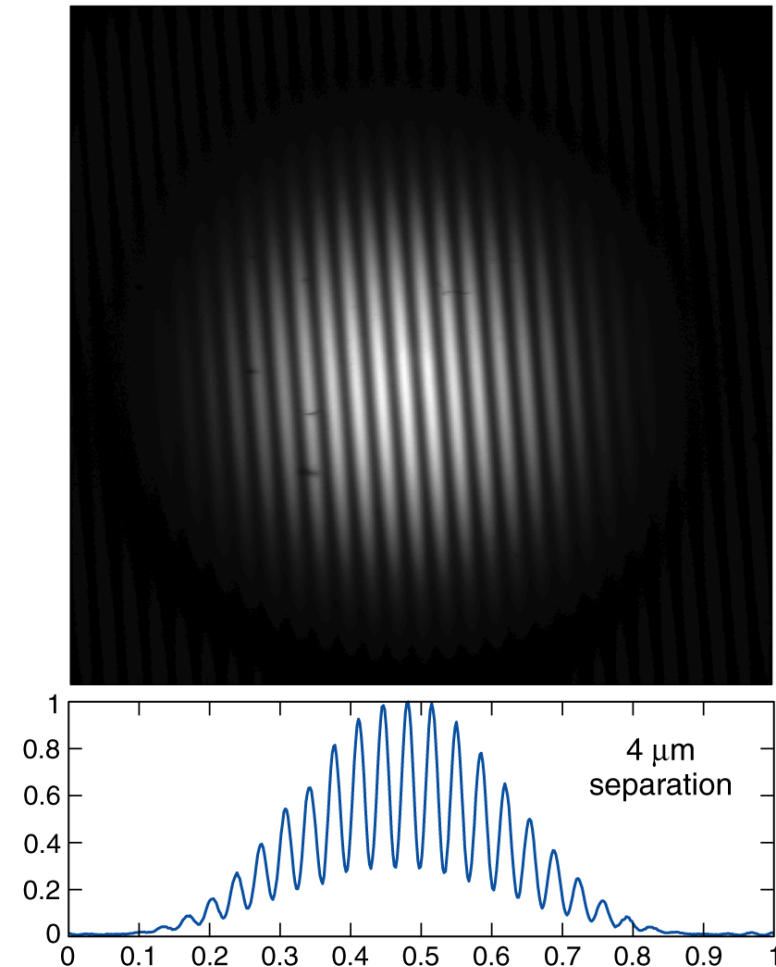
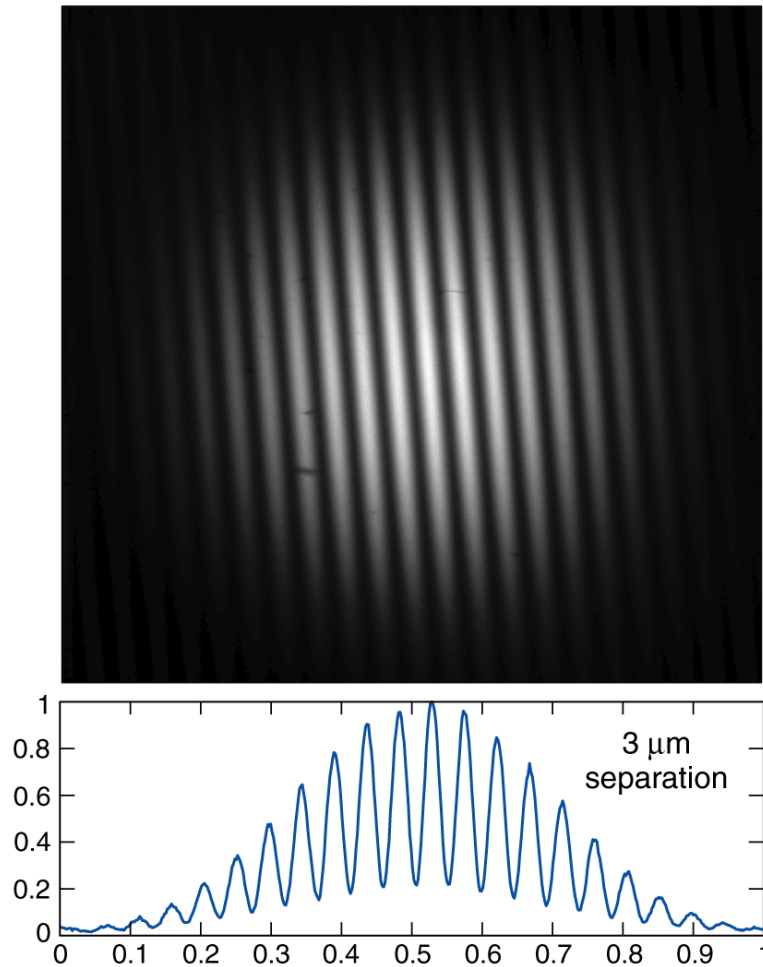
8 GeV, 100 mA
 $\lambda_u = 32$ mm, $N = 140$
 $0 \leq K \leq 2.46$
 $\sigma_x = 393$ μm , $\sigma_x' = 15.7$ μr
 $\sigma_y = 4.98$ μm , $\sigma_y' = 1.24$ μr
 $\theta_{\text{cen}} = 6.6$ μr
 $\eta = 10\%$



Spatial coherence and phase with Young's double slit interferometer



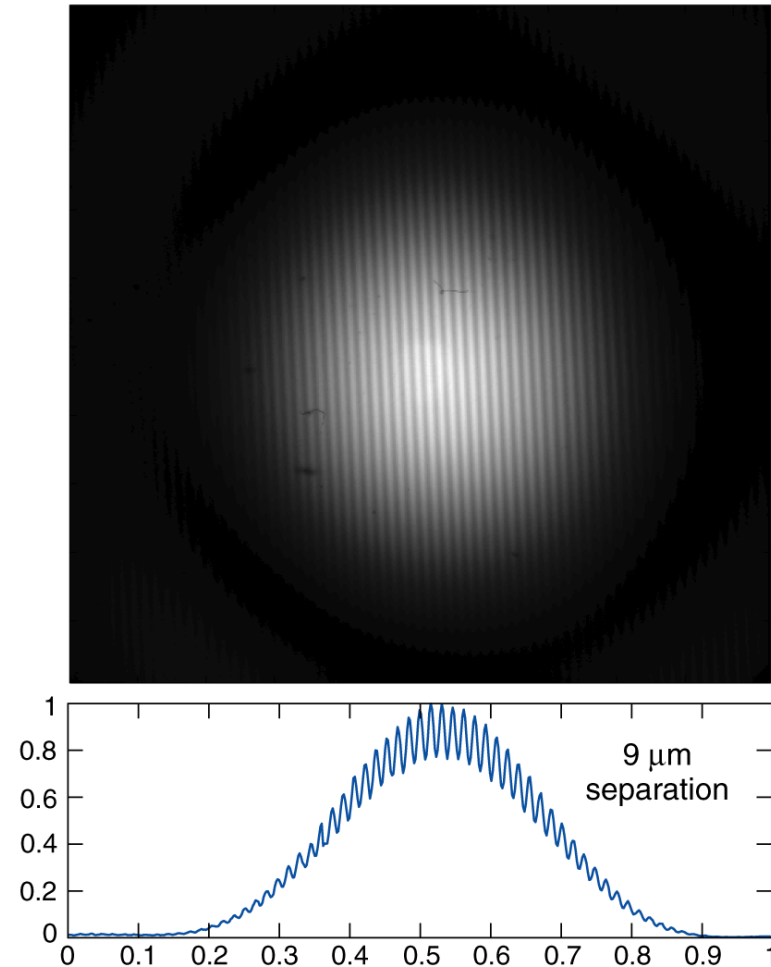
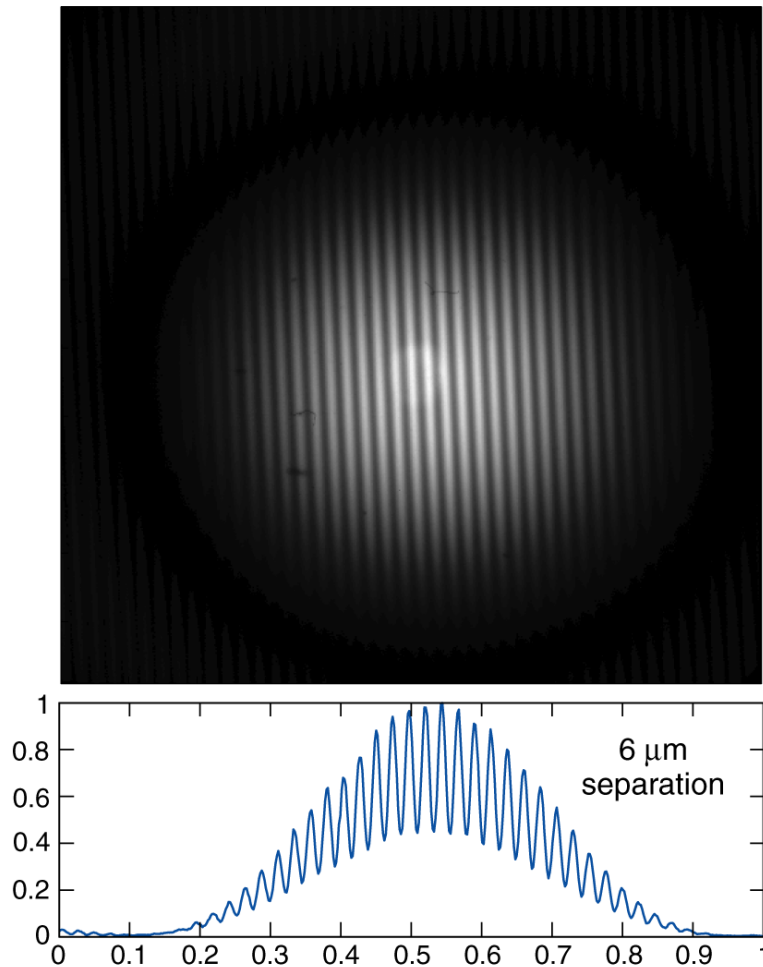
Spatial coherence measurements of undulator radiation using Young's 2-pinhole technique



Courtesy of Chang Chang, UC Berkeley and LBNL.

$\lambda = 13.4$ nm, 450 nm diameter pinholes, 1024 x 1024 EUV/CCD at 26 cm ALS, 1.9 GeV, $\lambda_u = 8$ cm, $N = 55$

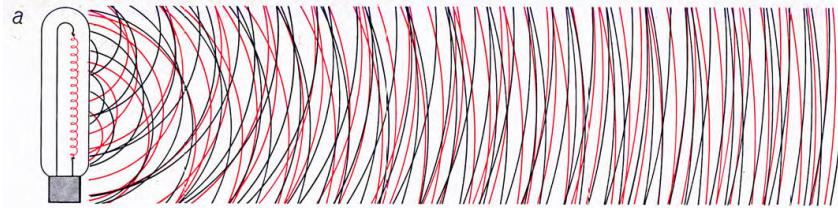
Spatial coherence measurements of undulator radiation using Young's 2-pinhole technique



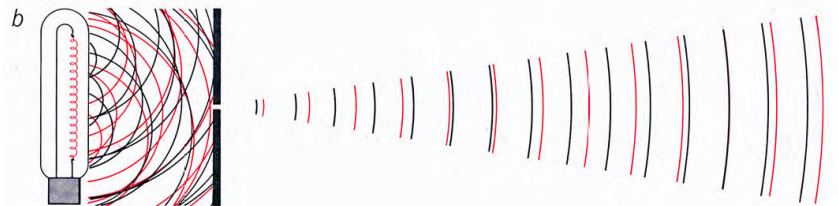
Courtesy of Chang Chang, UC Berkeley and LBNL.

$\lambda = 13.4$ nm, 450 nm diameter pinholes, 1024 x 1024 EUV/CCD at 26 cm ALS, 1.9 GeV, $\lambda_u = 8$ cm, $N = 55$

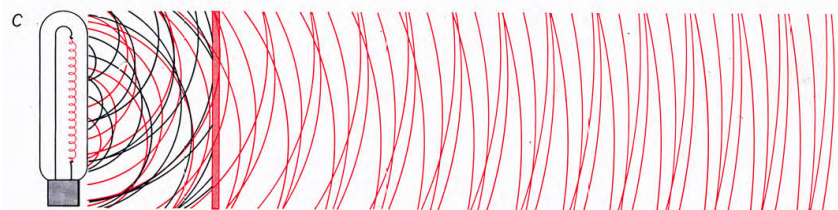
Ordinary light and laser light



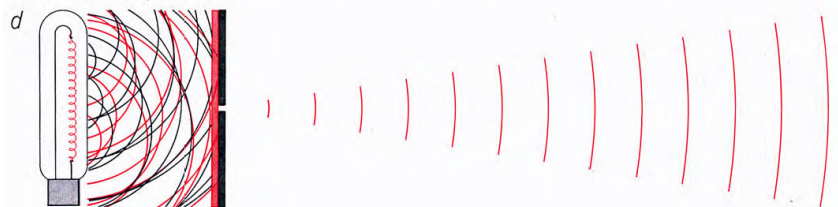
Ordinary thermal source, atoms radiate independently.



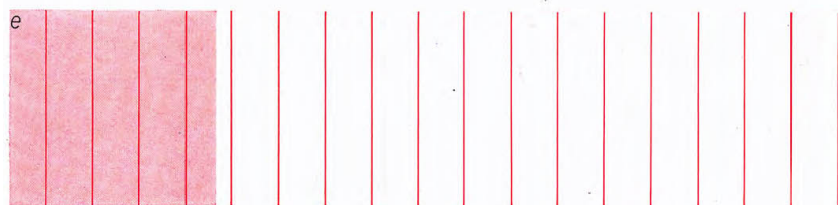
A pinhole can be used to obtain spatially coherent light, but at a great loss of power.



A color filter (or monochromator) can be used to obtain temporally coherent light, also at a great loss of power.



Pinhole and spectral filtering can be used to obtain light which is both spatially and temporally coherent but the power will be very small (tiny).



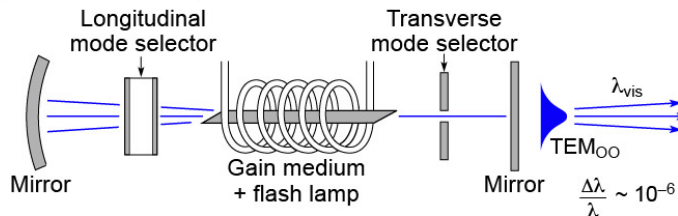
All of the laser light is both spatially and temporally coherent*.

Arthur Schawlow, "Laser Light", Sci. Amer. 219, 120 (Sept. 1968)

Coherence



Laser Cavity



Spatial and Temporal Coherence

$$d \cdot 2\theta \big|_{FWHM} \approx \frac{\lambda}{2}$$

$$\ell_{coh} = \lambda^2 / 2\Delta\lambda$$

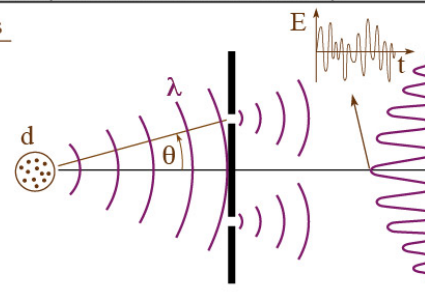
$$\tau_{coh} = \ell_{coh} / c$$

$$\Delta E \cdot \Delta \tau \big|_{FWHM} \geq 1.82 \text{ eV} \cdot \text{fsec}$$

Young's Double Slit (uncorrelated emitters)

N uncorrelated emitters

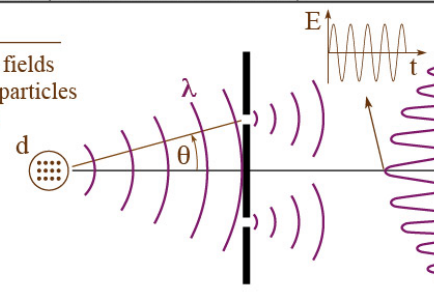
- Self-interference only
- Electric fields chaotic
- Intensities add
- Radiated power $\sim N$



Young's Double Slit (correlated emitters)

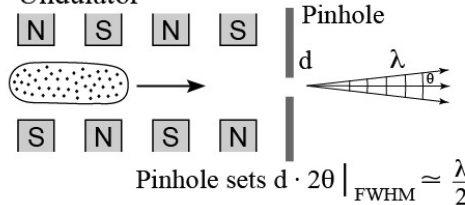
N correlated emitters

- Phase coherent electric fields
- Electric fields from all particles interfere constructively
- Radiated power $\sim N^2$

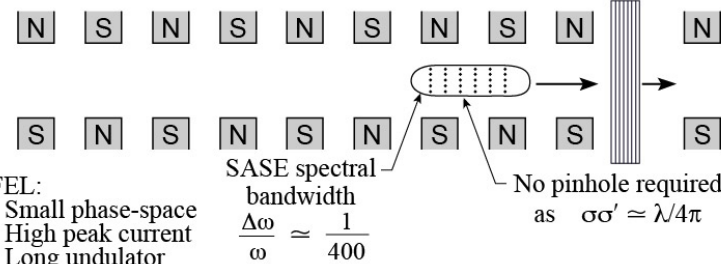


Undulators and Free-Electron Laser

Undulator



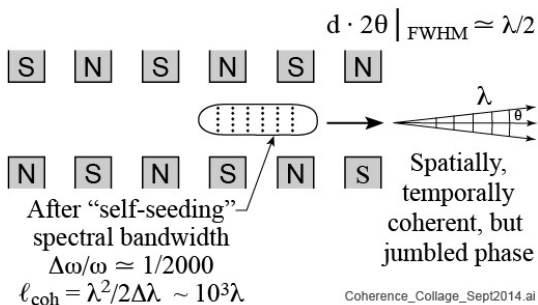
SASE FEL



- FEL:
- Small phase-space
 - High peak current
 - Long undulator

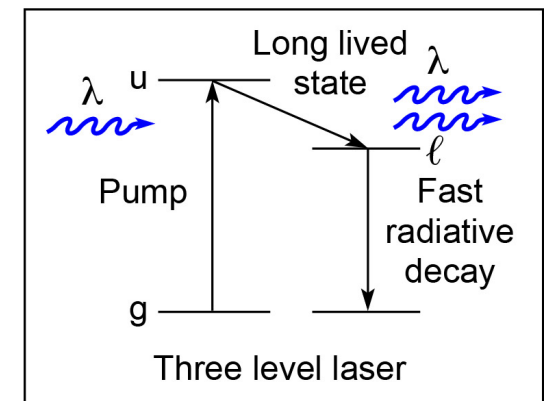
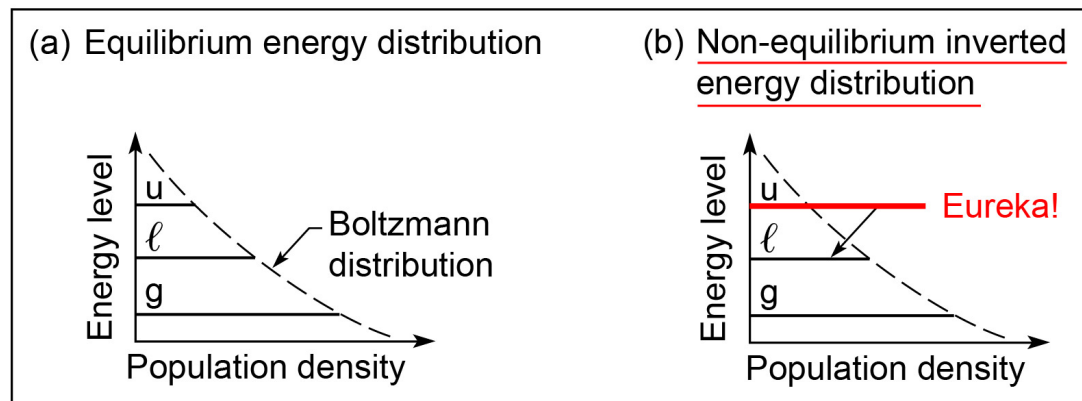
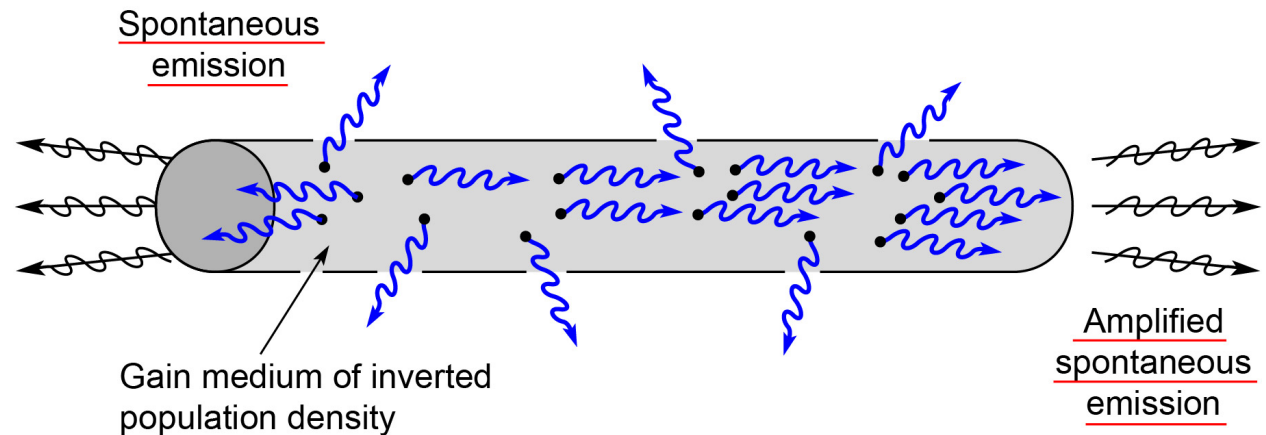
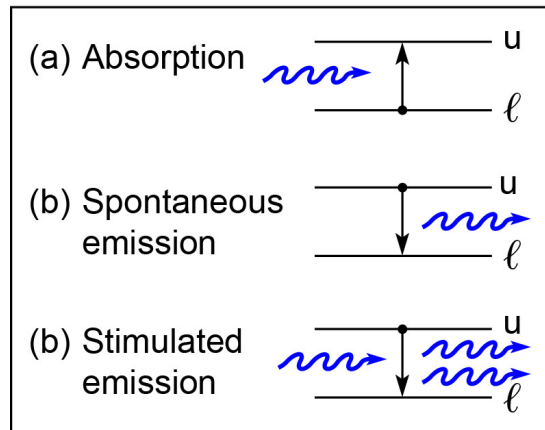
SASE spectral bandwidth
 $\frac{\Delta\omega}{\omega} \approx \frac{1}{400}$

Transmission x-ray crystal
 No pinhole required
 as $\sigma\sigma' \approx \lambda/4\pi$



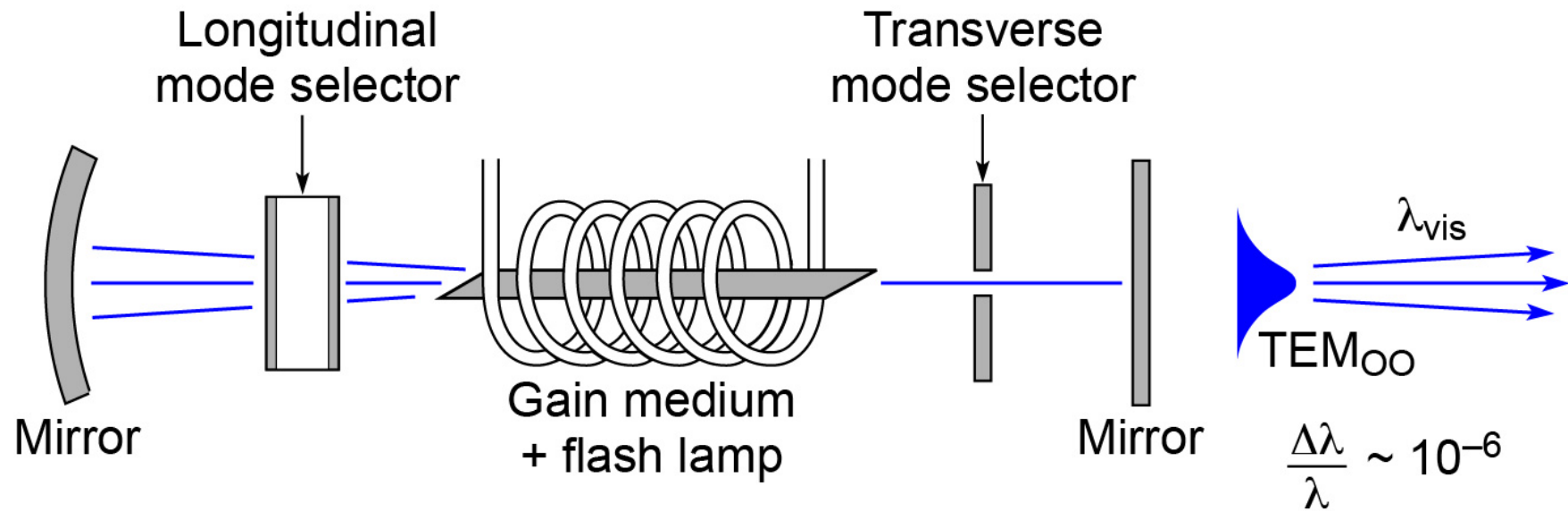
Coherence_Collage_Sept2014.ai

Amplified Spontaneous Emission (ASE)





A visible light laser cavity



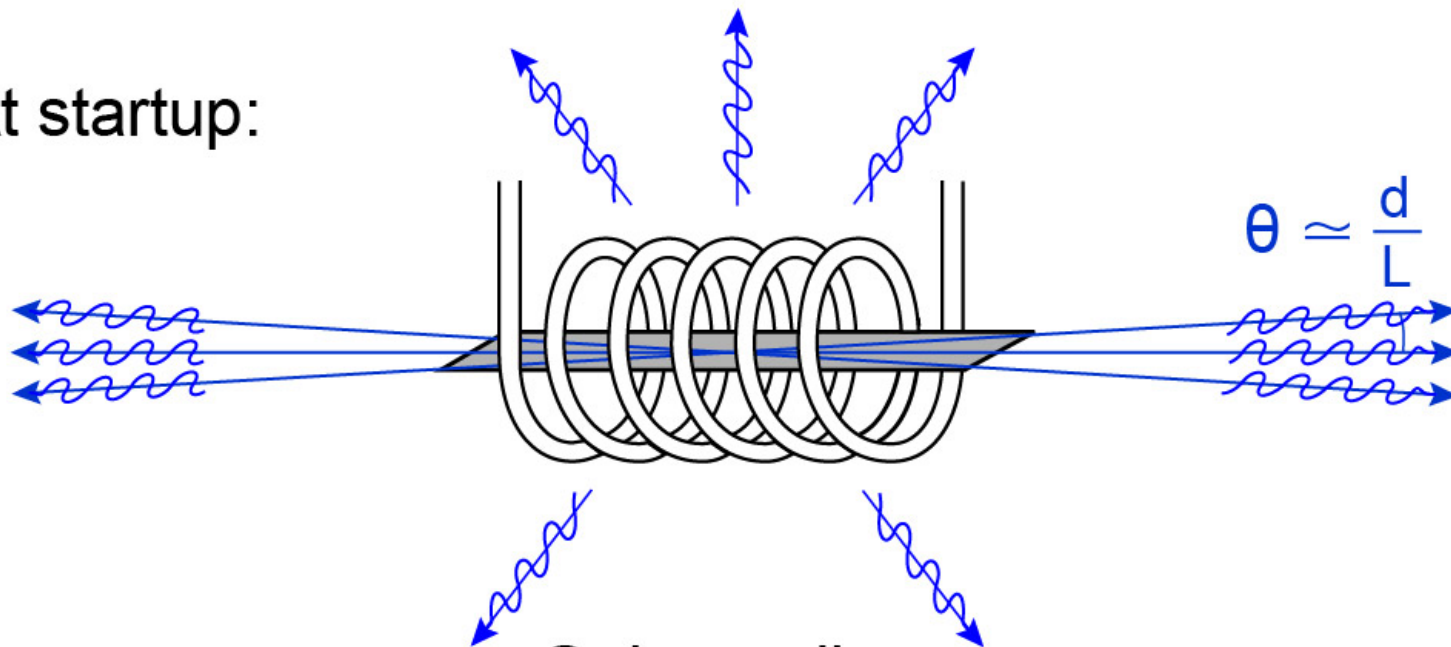
Ch07_06_Sept2013.ai



Building a laser cavity

- (a) Amplified spontaneous emission
in a long active medium (rod)

At startup:

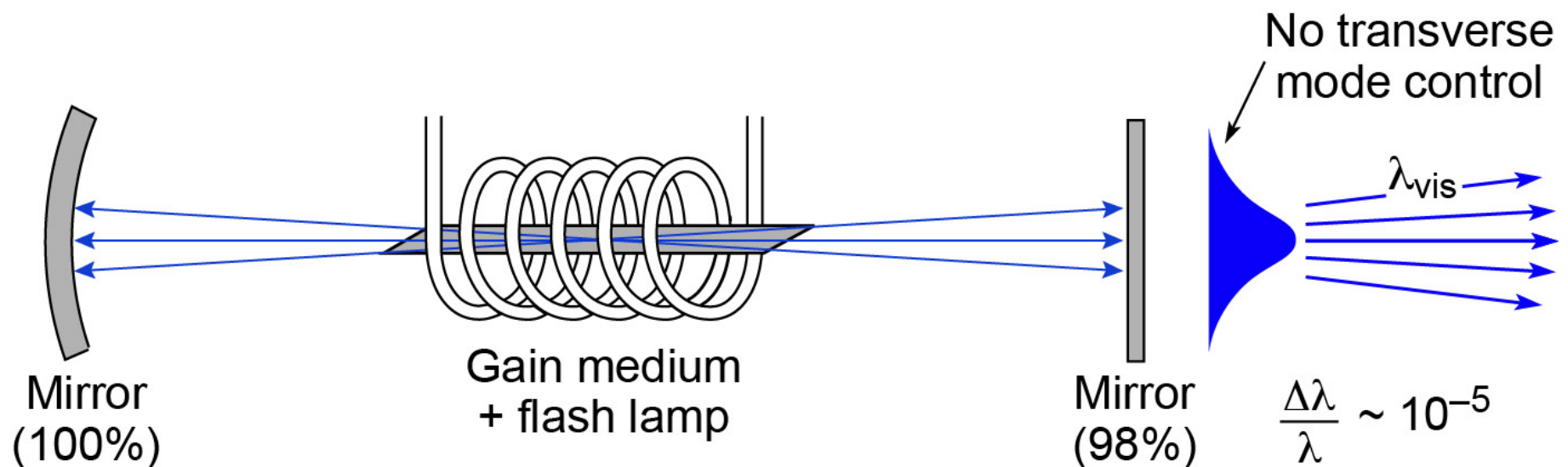


Gain medium
+ flash lamp
(drives population
inversion)

Building a laser cavity

(b)

Cavity end-mirrors for feedback
and enhanced growth

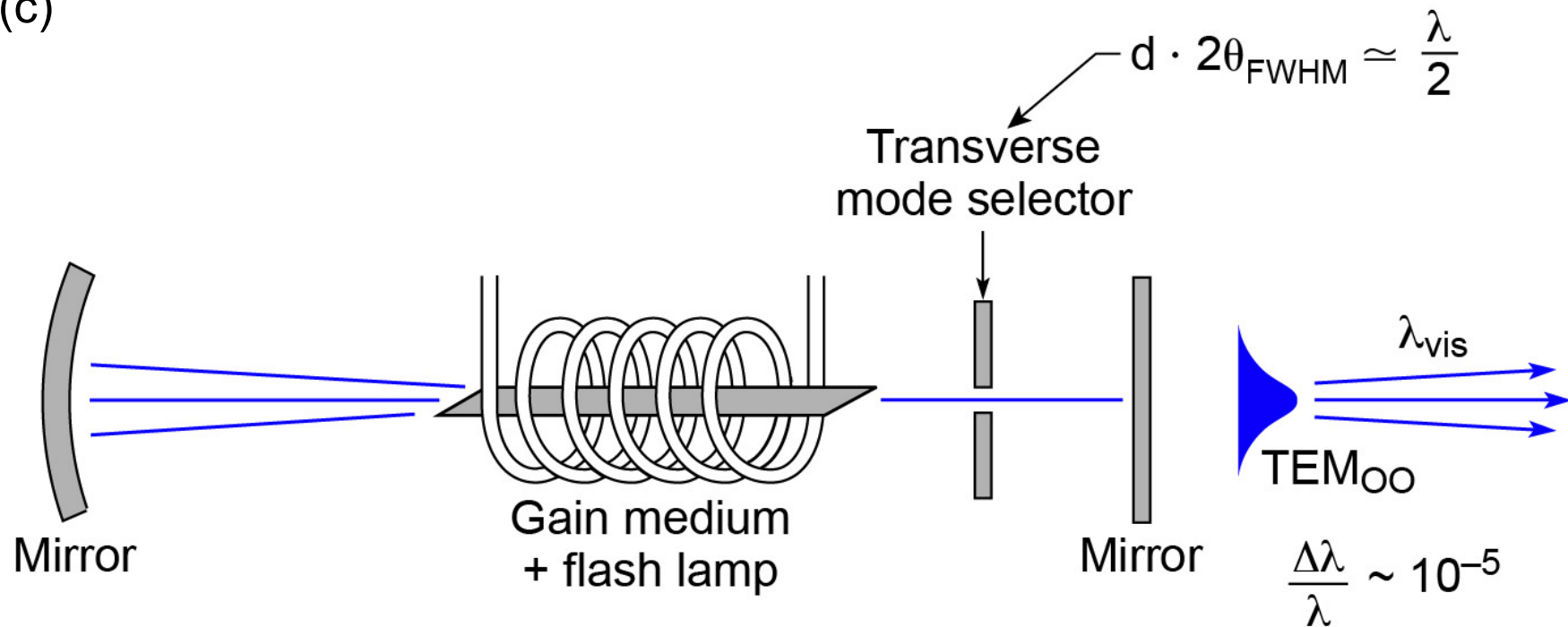


- Increased output; many gain lengths
- Greatly reduced side losses
- $\theta = \frac{d}{L} \sim 1 \text{ mrad}$; 1000 x diffraction limited

Improving spatial coherence



(c)

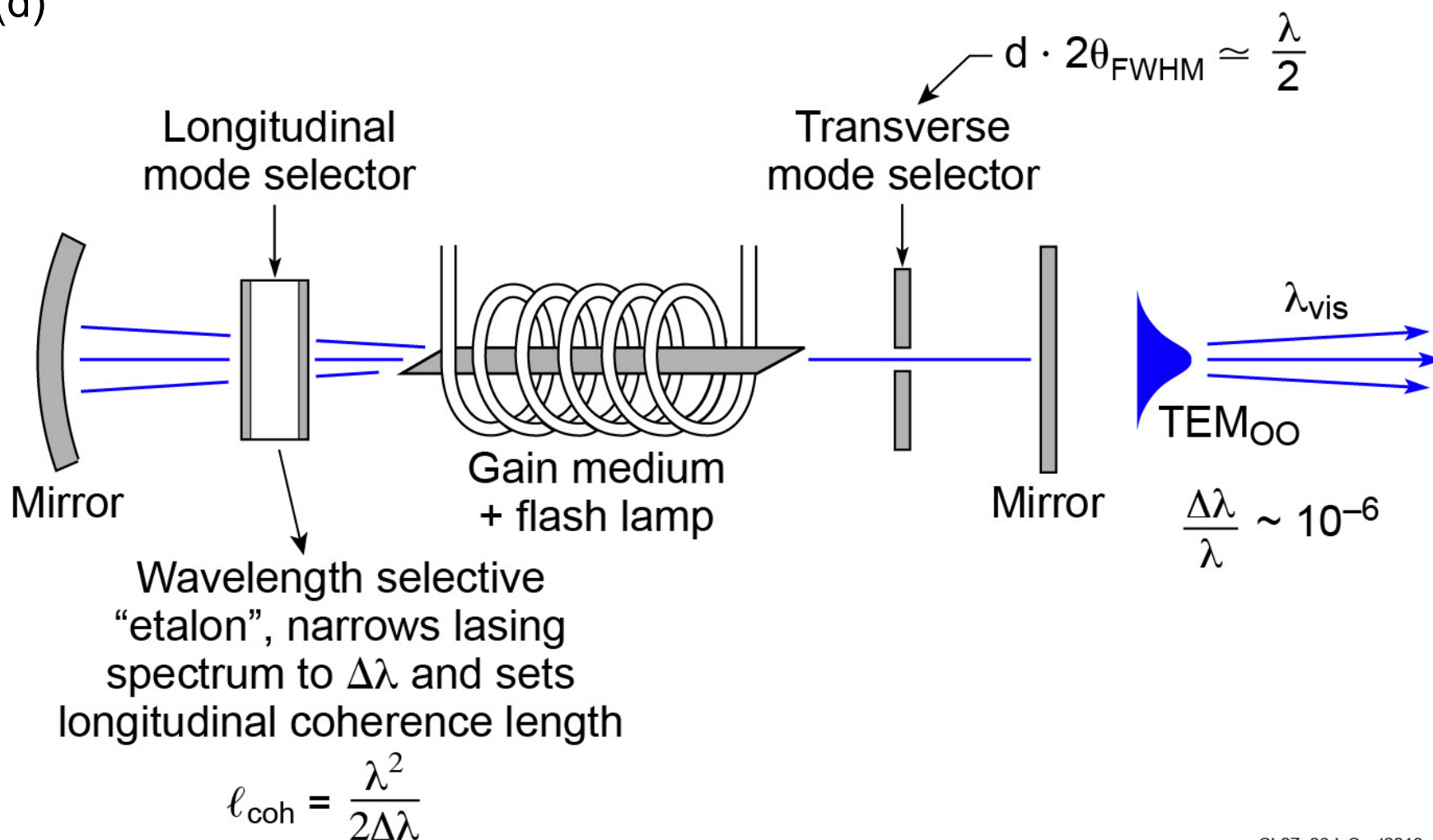


Ch07_06c_June2014.ai

Building a laser cavity for full spatial and temporal coherence



(d)



Ch07_06d_Sept2013.ai

The Nobel Prize in Physics 1964



Charles Hard Townes



Nicolay Gennadiyevich Basov



Aleksandr Mikhailovich Prokhorov

“For fundamental work in the field of quantum electronics, which has led to the construction of oscillators and amplifiers based on the maser-laser principle”

The Nobel Prize in Physics 1981



Nicolaas Bloembergen

*“Non-linear optics
and spectroscopy”*



Arthur Leonard Schawlow

*“Spectroscopy in a new
light”*

**“For contributions to the development
of laser spectroscopy”**



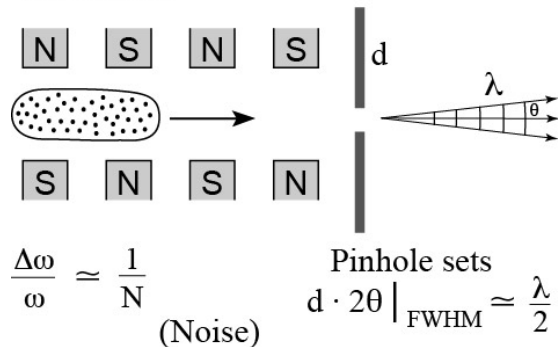
Undulators, FELs and coherence

- Spatial coherence
- Temporal coherence
- Partial coherence
- Full coherence
- Spatial filtering
- Uncorrelated emitters
- Correlated emitters
- True phase coherence and mode control
- Lasers, amplified spontaneous emission (ASE) and mode control
- Undulator radiation
- SASE FEL 10-70 fsec EUV/x-rays
- Seeded FEL true phase coherent x-rays
- High harmonic generation (HHG) compact fsec/asec EUV
- EUV lasers and laser seeded HHG
- Applications with uncorrelated emitters
- Applications with correlated emitters

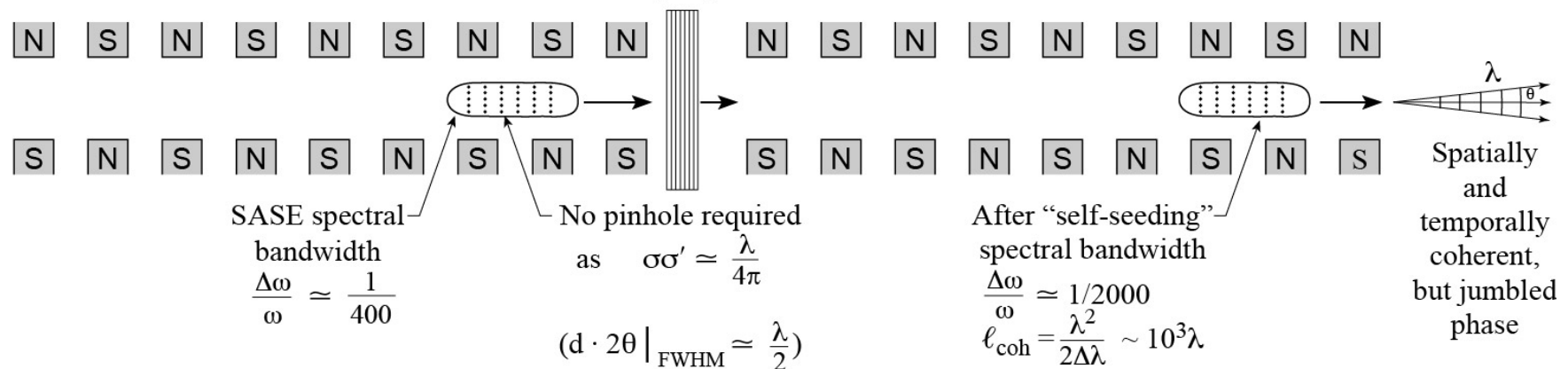
Spatial and temporal coherence with undulators and FELs



Undulator



SASE FEL



FEL:

- Small phase-space
- High peak current
- Long undulator

Future possibilities:

- Is a phase-coherent seed possible at x-ray wavelengths?
- Perhaps a seed plus HGHG multiplication, as at FERMI?
- Is an x-ray FEL oscillator possible?

The evolution of incoherent clapping (applauding) to coherent clapping



Suggested by Hideo Kitamura,
(RIKEN)



The bunching advantage of FELs

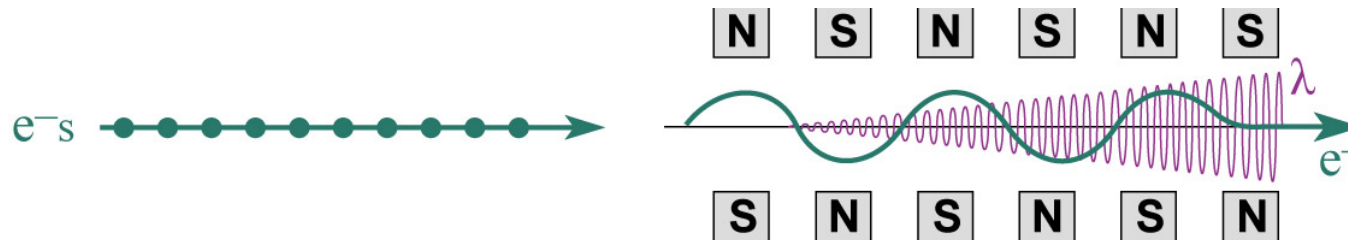
In an undulator with random, uncorrelated electron positions within the bunch, only the radiated self-fields **E** add constructively.

- Coherence is somewhat limited
- Power radiated is proportional to N_e (total # electrons)

For FEL lasing the radiated fields are strong enough to form “microbunches” within which the electron positions are well correlated. Radiated fields from these correlated electrons are in phase. The net electric field scales with N_{ej} , the # of electrons in the microbunch, and power scales with N_{ej}^2 times the number of microbunches, n_j .

- Essentially full spatial coherence
- Power radiated is proportional to $N_{ej}^2 n_j \sim 10^6 \times 300 \sim 3 \times 10^8$

FEL Physics



- Uniformly distributed particles (beam) into undulator.
- Emission of radiation (“spontaneous” emission).
- Wave grows enough (undulator radiation) to begin affecting particle dynamics through $m\mathbf{a} = -e\mathbf{E}$ radiation.
- Transverse coupling between \mathbf{E}_{rad} and transverse velocity \mathbf{v}_x (in undulator) leads to energy exchange between fields and particle (zero net at first) $\frac{dE_e}{dt} = mc^2 \frac{d\gamma}{dt} = \mathbf{F} \cdot \mathbf{v} = -e \mathbf{E} \cdot \mathbf{v}_x$
- Modulated velocities with increments in \mathbf{v}_x lead to bunching on axis.
- Electron density modulation leads to stronger radiation,

$$P_{\text{rad}} \propto q^2 |a|^2 \sim (eN_e)^2 \cdot \frac{e^2}{m^2} = N_e^2 \frac{e^4}{m^2}$$
- Stronger fields (wave) drive stronger transverse velocity.
- Stronger \mathbf{v}_x drives stronger bunching, . . . stronger fields, . . . FEL action.

Equations of motion for the stronger electric field FEL



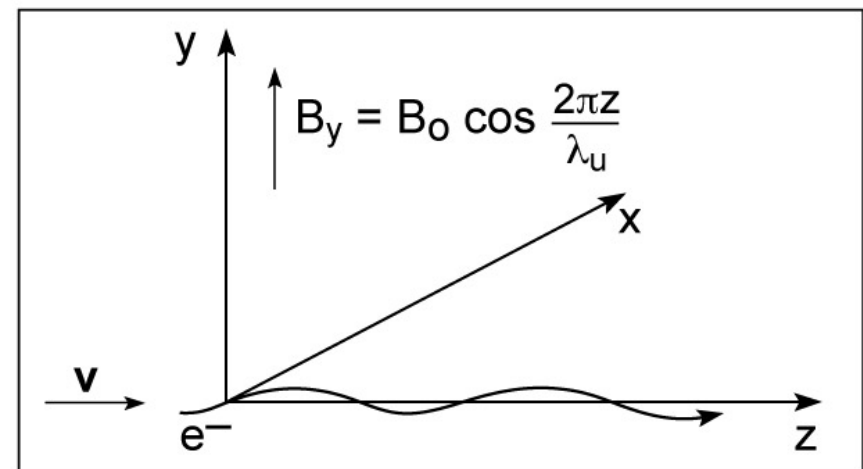
Magnetic fields in the periodic undulator cause the electrons to oscillate and thus radiate. In the FEL case the undulator is very long, the radiated electric field is now strong enough that it can not be ignored in the momentum equation, and the energy of the electrons is no longer constant.

Equations of motion:

$$\frac{d\mathbf{p}}{dt} = -e(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

$$\frac{dE_e}{dt} = \mathbf{v} \cdot \mathbf{F} = -e\mathbf{v} \cdot \mathbf{E} = -\frac{eKE}{2mc\gamma} \sin\phi_s$$

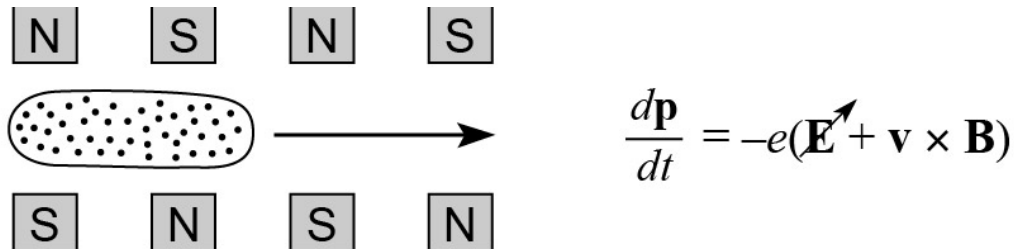
where $\mathbf{p} = \gamma m\mathbf{v}$ and $E_e = m\gamma c^2$.



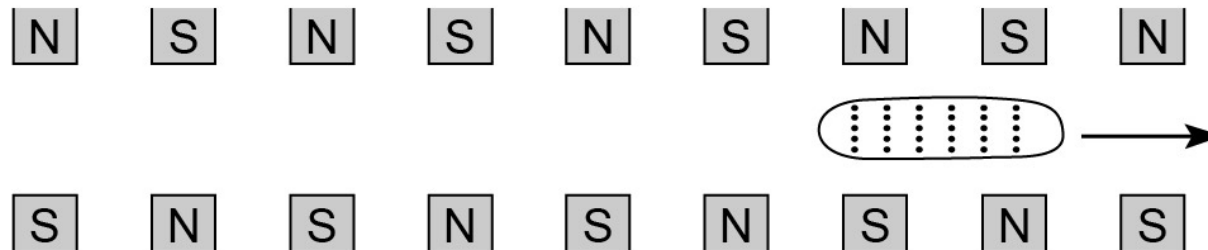
Ch05_F15_Eq16_19.top_Nov2011.ai



Undulators and FELs



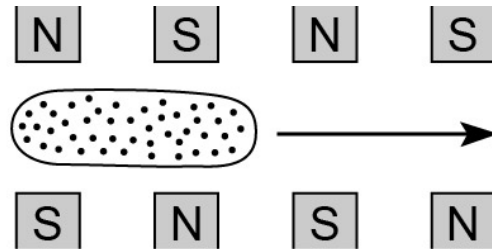
Undulator – uncorrelated electron positions, radiated fields uncorrelated, intensities add, limited coherence, power $\sim N$.



Free Electron Laser (FEL) – very long undulator, electrons are “microbunched” by their own radiated fields into strongly correlated waves of electrons, all radiated electric fields now add, spatially coherent, power $\sim N^2$

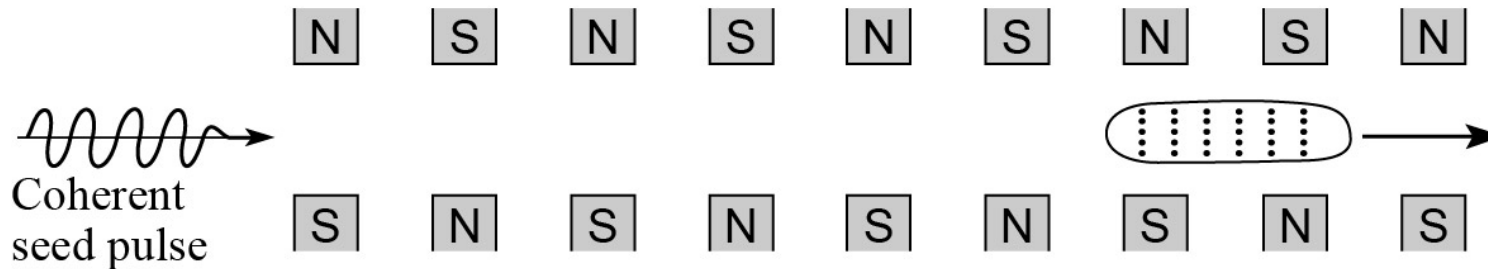


Seeded FEL



$$\frac{d\mathbf{p}}{dt} = -e(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

Undulator – uncorrelated electron positions, radiated fields uncorrelated, intensities add, limited coherence, power $\sim N$.

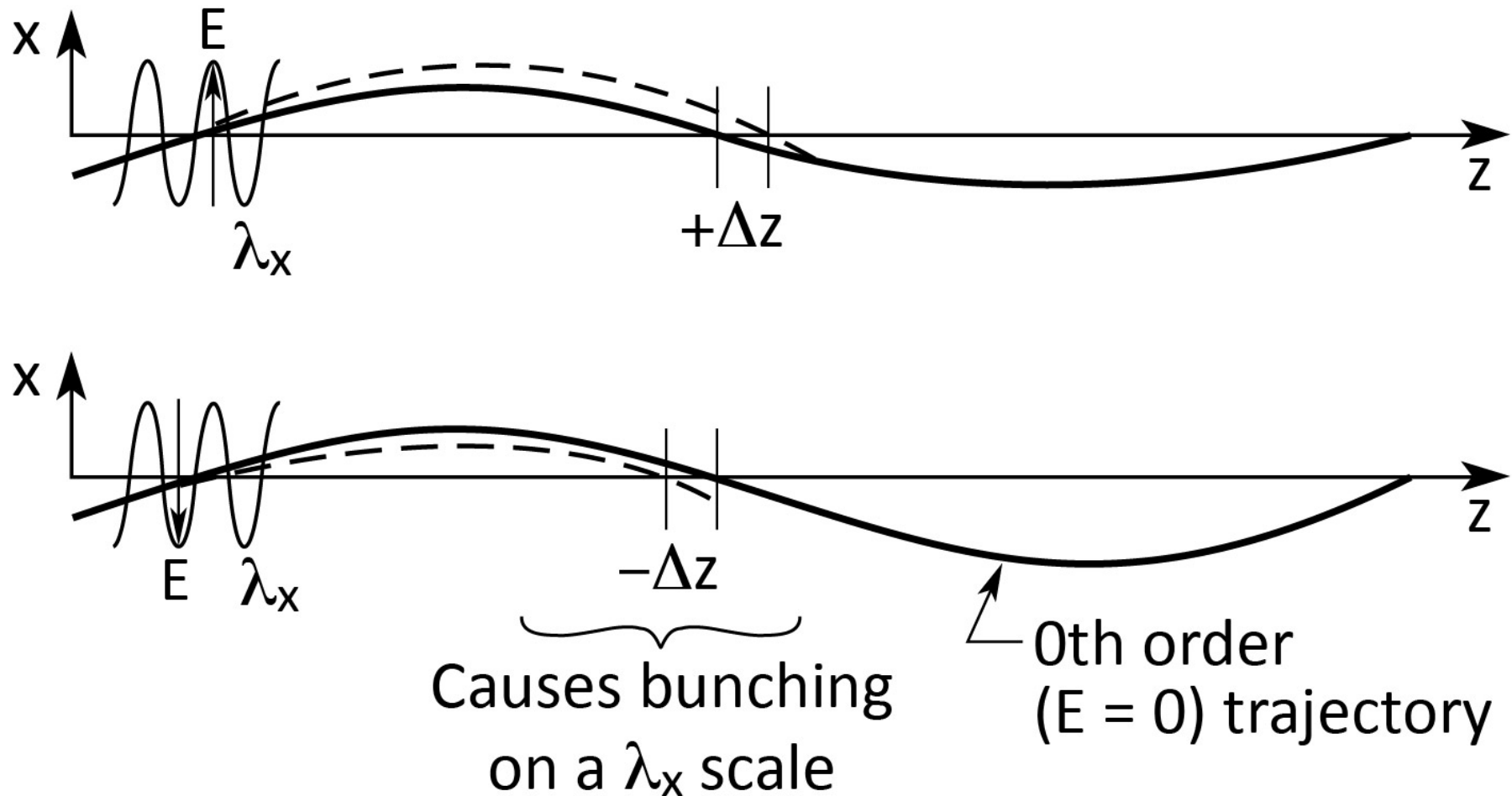


Free Electron Laser (FEL) – very long undulator, electrons are “microbunched” by their own radiated fields into strongly correlated waves of electrons, all radiated electric fields now add, spatially coherent, power $\sim N^2$

$$\frac{d\mathbf{p}}{dt} = -e(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

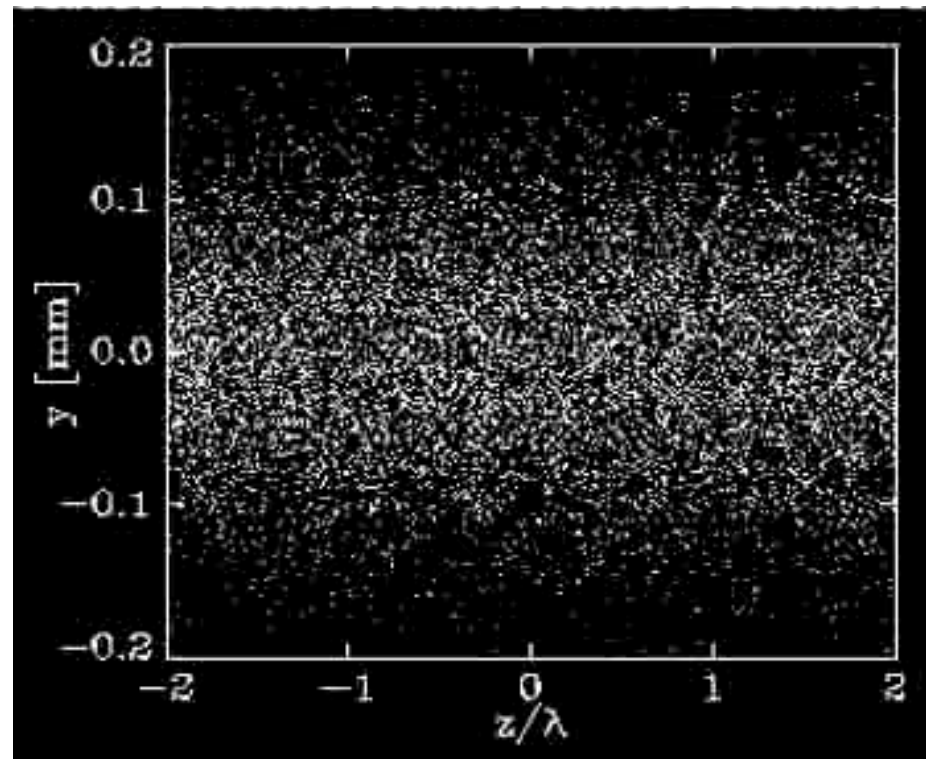
Seeded FEL. Initial bunching driven by phase coherent seed laser pulse. Improved pulse structure and spectrum.

Electron energies and subsequent axis crossings are affected by the amplitude and relative phase of the co-propagating field



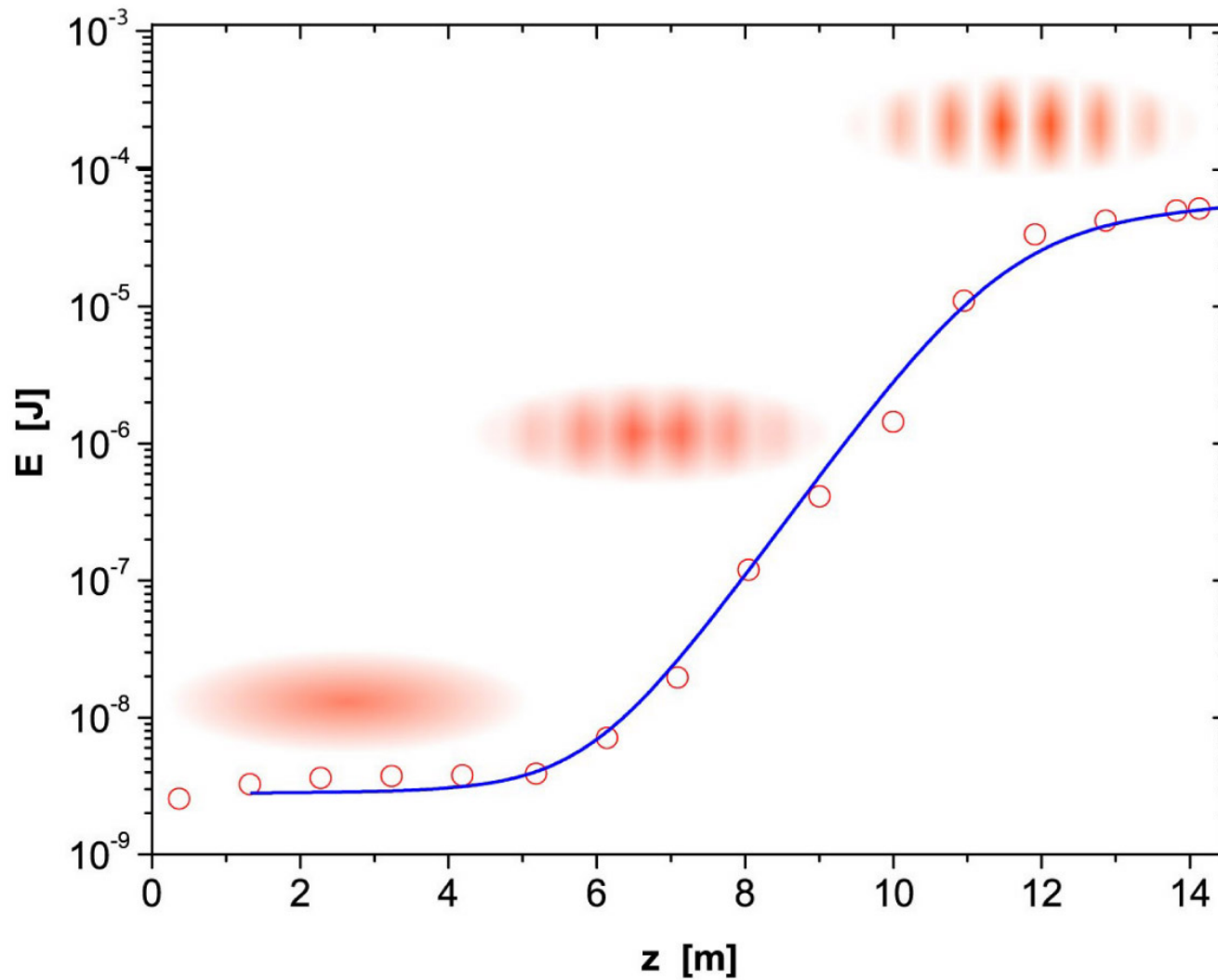
FEL_Prebunching.ai

FEL Microbunching



Courtesy of Sven Reiche, UCLA, now SLS

Gain and saturation in an FEL



Courtesy of K-J. Kim

Gain_Saturation_FEL_graph.ai



FEL lasing and the parameter ρ_{FEL}

Equations of motion:

$$\frac{d\mathbf{p}}{dt} = \frac{d}{dt}(\gamma m \mathbf{v}) = -e(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

$$\frac{dE_e}{dt} = mc^2 \frac{d\gamma}{dt} = \mathbf{v} \cdot \mathbf{F} = -e \mathbf{v} \cdot \mathbf{E} = -\frac{e \hat{K} E}{2mc\gamma} \sin\phi_s \quad ; \quad \hat{K} = K [\text{JJ}]$$

Strength of wave-particle coupling:

$$\rho_{\text{FEL}} = \left(\frac{\hat{K}^2 r_e n_e \lambda_u^2}{32\pi\gamma^3} \right)^{1/3}$$

Lasing gain length (power e-folding):

$$L_{\text{Gain}} = \lambda_u / 4\pi\sqrt{3} \rho_{\text{FEL}}$$

Saturated lasing power:

$$\hat{P}_{\text{sat}} \approx \rho_{\text{FEL}} \cdot \gamma mc^2 \cdot \hat{I}/e$$

Limited electron energy spread for efficient energy transfer:

$$\sigma_\gamma/\gamma < \rho_{\text{FEL}}$$

Electron angular limit for constructive interference:

$$\sigma' \leq 1/\gamma \sqrt{N}$$

Electron phase-space (“emittance”) match to coherent radiation phase-space:

$$\sigma\sigma' \leq \lambda/4\pi$$

Relative spectral bandwidth:

$$\Delta\lambda/\lambda \sim \rho_{\text{FEL}}$$

First lasing and operation of an ångstrom-wavelength free-electron laser

P. Emma^{1*}, R. Akre¹, J. Arthur¹, R. Bionta², C. Bostedt¹, J. Bozek¹, A. Brachmann¹, P. Bucksbaum¹, R. Coffee¹, F.-J. Decker¹, Y. Ding¹, D. Dowell¹, S. Edstrom¹, A. Fisher¹, J. Frisch¹, S. Gilevich¹, J. Hastings¹, G. Hays¹, Ph. Hering¹, Z. Huang¹, R. Iverson¹, H. Loos¹, M. Messerschmidt¹, A. Miahnahri¹, S. Moeller¹, H.-D. Nuhn¹, G. Pile³, D. Ratner¹, J. Rzepiela¹, D. Schultz¹, T. Smith¹, P. Stefan¹, H. Tompkins¹, J. Turner¹, J. Welch¹, W. White¹, J. Wu¹, G. Yocky¹ and J. Galayda¹

The recently commissioned Linac Coherent Light Source is an X-ray free-electron laser at the SLAC National Accelerator Laboratory. It produces coherent soft and hard X-rays with peak brightness nearly ten orders of magnitude beyond conventional synchrotron sources and a range of pulse durations from 500 to <10 fs (10^{-15} s). With these beam characteristics this light source is capable of imaging the structure and dynamics of matter at atomic size and timescales. The facility is now operating at X-ray wavelengths from 22 to 1.2 Å and is presently delivering this high-brilliance beam to a growing array of scientific researchers. We describe the operation and performance of this new 'fourth-generation light source'.

Over the past several decades, intense beams of X-rays from synchrotron light sources have been used quite successfully to reveal the arrangement of atoms in a wide range of materials, including semiconductors, polymers, ceramics and biological molecules. Unlike visible light, X-ray wavelengths are comparable to the interatomic spacings in matter, thus enabling imaging on the atomic scale. However, even so-called 'third-generation' synchrotron sources produce insufficient brightness to resolve the structure of isolated single molecules. Synchrotrons also generate picoseconds-long X-ray pulses, which yield only blurred images of atoms and molecules in motion. In addition, accelerator research¹ has been focused for some time on advancements in applications of accelerators towards coherent emission from a bunch of electrons. These motivations, along with the desire to invent an X-ray laser, have led to the development of a 'fourth-generation light source' based on the free-electron laser (FEL).

A FEL² is a source of coherent radiation that uses the interaction between an electromagnetic wave and a bunch of relativistic electrons to amplify this wave as the electrons pass through a periodic magnetic array (an undulator). In principle, the FEL wavelength is

length with power saturation after 18–20 gain lengths, providing stability of the shot-to-shot FEL power against small variations of the electron beam brightness.

The Linac Coherent Light Source (LCLS)^{6,7}, first proposed⁸ in 1992, is such a light source, with ultrafast pulse durations on the femtosecond timescale⁹. It allows the imaging of single molecules and, working like a high-speed camera, also enables the creation of molecular movies, potentially revealing the details of physical, chemical and biological dynamics on an unprecedented timescale¹⁰.

The principles of the SASE FEL have been demonstrated at longer wavelengths^{11–16}, and the FLASH facility in Hamburg in Germany currently operates such a SASE FEL at wavelengths down to 65 Å (ref. 17). Several hard-X-ray machines have either been proposed or are under construction^{18–20}, but the LCLS is the first hard-X-ray FEL in operation and has served users since 1 October 2009. The performance and operation of the LCLS are described here, beginning with a list of typical measured and design parameters (Table 1) for both soft and hard X-ray settings. The X-ray wavelength is easily tunable from 22 to 1.2 Å by varying the electron energy in the range 3.5–15 GeV.

(LCLS, lasing April 2009, 1st day; saturated lasing 2009; publ. Sept. 2010)

¹SLAC National Accelerator Laboratory, Stanford, California 94309, USA, ²Lawrence Livermore National Laboratory, Livermore, California 94550, USA,

³Argonne National Laboratory, Argonne, Illinois 60439, USA. *e-mail: emma@slac.stanford.edu

Table 1 | Design and typical measured parameters for both hard (8.3 keV) and soft (0.8–2.0 keV) X-rays. The ‘design’ and ‘hard’ values are shown only at 8.3 keV. Stability levels are measured over a few minutes.

Parameter	Design	Hard	Soft	Unit
Electrons				
Charge per bunch	1	0.25	0.25	nC
Single bunch repetition rate	120	30	30	Hz
Final linac e^- energy	13.6	13.6	3.5–6.7	GeV
Slice [†] emittance (injected)	1.2	0.4	0.4	μm
Final projected [†] emittance	1.5	0.5–1.2	0.5–1.6	μm
Final peak current	3.4	2.5–3.5	0.5–3.5	kA
Timing stability (r.m.s.)	120	50	50	fs
Peak current stability (r.m.s.)	12	8–12	5–10	%
X-rays				
FEL gain length	4.4	3.5	~1.5	m
Radiation wavelength	1.5	1.5	6–22	\AA
Photons per pulse	2.0	1.0–2.3	10–20	10^{12}
Energy in X-ray pulse	1.5	1.5–3.0	1–2.5	mJ
Peak X-ray power	10	15–40	3–35	GW
Pulse length (FWHM)	200	70–100	70–500	fs
Bandwidth (FWHM)	0.1	0.2–0.5	0.2–1.0	%
Peak brightness (estimated)	8	20	0.3	10^{32} *
Wavelength stability (r.m.s.)	0.2	0.1	0.2	%
Power stability (r.m.s.)	20	5–12	3–10	%

*Brightness is photons per phase space volume, or photons $\text{s}^{-1} \text{mm}^{-2} \text{mrad}^{-2}$ per 0.1% spectral bandwidth.

[†]Slice[†] refers to femtosecond-scale time slices and ‘projected’ to the full time-projected (that is, integrated) emittance of the bunch.

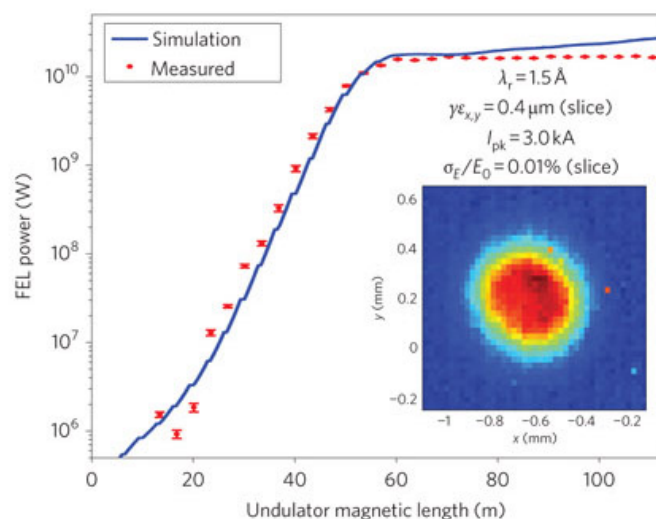


Figure 4 | FEL gain length measurement at 1.5 Å. Measured FEL power (red points) plotted after continuous insertion of each 3.4-m undulator segment showing saturation at 60 m and with all 33 undulator segments installed. Error bars represent the r.m.s. statistical uncertainty in the measured power when averaging 30 beam pulses. The measured gain length is 3.5 m with a GENESIS simulation overlaid (blue curve) and with consistent electron beam parameters shown. The YAG screen image is shown in the inset with 140- μm r.m.s. round X-ray spot size in this early case (April 2009). λ_r is the fundamental FEL radiation wavelength; I_{pk} is the peak current of the electron beam in the undulator; γ is the relativistic Lorentz factor; $\epsilon_{x,y}$ is the transverse r.m.s. emittance of the electron beam in the undulator; $\gamma\epsilon_{x,y}$ is the normalized transverse r.m.s. emittance of the electron beam in the undulator; σ_E/E_0 is the r.m.s. relative energy spread of the electron beam in the undulator (that is, the r.m.s. energy spread, σ_E , divided by the mean electron energy, E_0).



Stanford's LCLS Free Electron Laser

Electron beam

$$E_e = 13.6 \text{ GeV}$$

$$\gamma = 26,600$$

$$\hat{I} = 3.4 \text{ kA}$$

$$\epsilon_n = \gamma\epsilon = 1.2 \text{ mm} \cdot \text{mrad}$$

$$Q = \frac{1}{4} \text{ nC} \quad (\sim 10^9 \text{ e}^- \text{s})$$

$$R_{\text{ep}} = 30 - 120 \text{ Hz}$$

$$\sigma_\gamma/\gamma = 1 \times 10^{-4}$$

$$\left. \begin{array}{l} 2.35\sigma = 78 \text{ } \mu\text{m} \\ 2.35\sigma' = 1 \text{ } \mu\text{r} \end{array} \right\} \sigma\sigma' = 14 \text{ pm} \cdot \text{rad} \quad \therefore \approx 1.2 \text{ diff. ltd.} \quad @ 1.5 \text{ } \text{\AA}$$

$$2.35\sigma_z = 21 \text{ } \mu\text{m FWHM}$$

$$(70 \text{ fsec @ } 300 \text{ nm/fsec})$$

Undulator

$$\lambda_u = 3 \text{ cm}$$

$$N = 33 \times 113 = 3733$$

$$L_{\text{magnetic}} = 3733 \times 3 \text{ cm} = 112 \text{ m}$$

$$K = 3.5 @ \lambda = 1.5 \text{ } \text{\AA}$$

Photons

$$\lambda = 1.5 \text{ } \text{\AA} \quad (8.2 \text{ keV})$$

$$F = 0.84 \times 10^{12} \text{ ph/pulse}$$

$$\hat{E} = 1.1 \text{ mJ/pulse}$$

$$\Delta\tau_{\text{ph}} = 10\text{-}70 \text{ fsec FWHM}$$

$$\lambda/\Delta\lambda = 200\text{-}500 \text{ FWHM}$$

$$\hat{P} = 15 \text{ GW}$$

FEL parameters

$$\rho_{\text{FEL}} = \left(\frac{\hat{K}^2 r_e n_e \lambda_u^2}{32\pi\gamma^3} \right)^{1/3} = 4.7 \times 10^{-4}$$

$$\hat{P}_{\text{sat}} = \rho_{\text{FEL}} \cdot \gamma mc^2 \cdot \hat{I}/e = 20 \text{ GW}$$

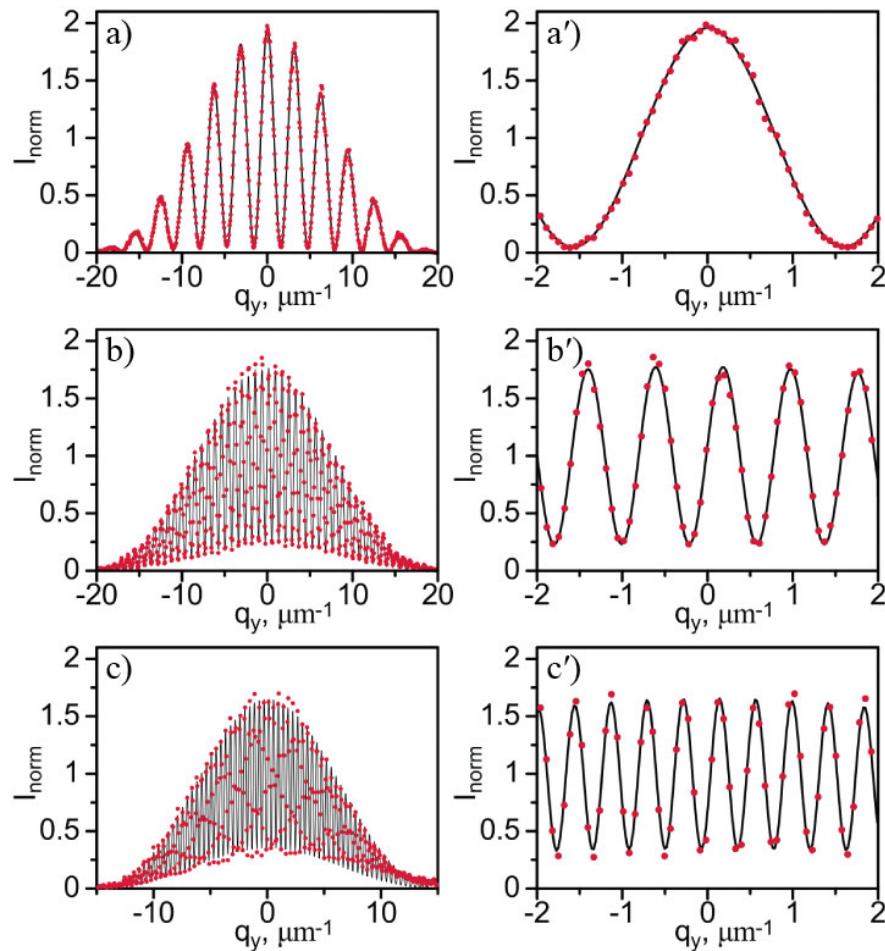
$$L_G = \lambda_u/4\pi\sqrt{3} \rho_{\text{FEL}} = 3.5 \text{ m}$$

$$L_{\text{sat}} = 60 \text{ m (of } 112 \text{ m)}$$

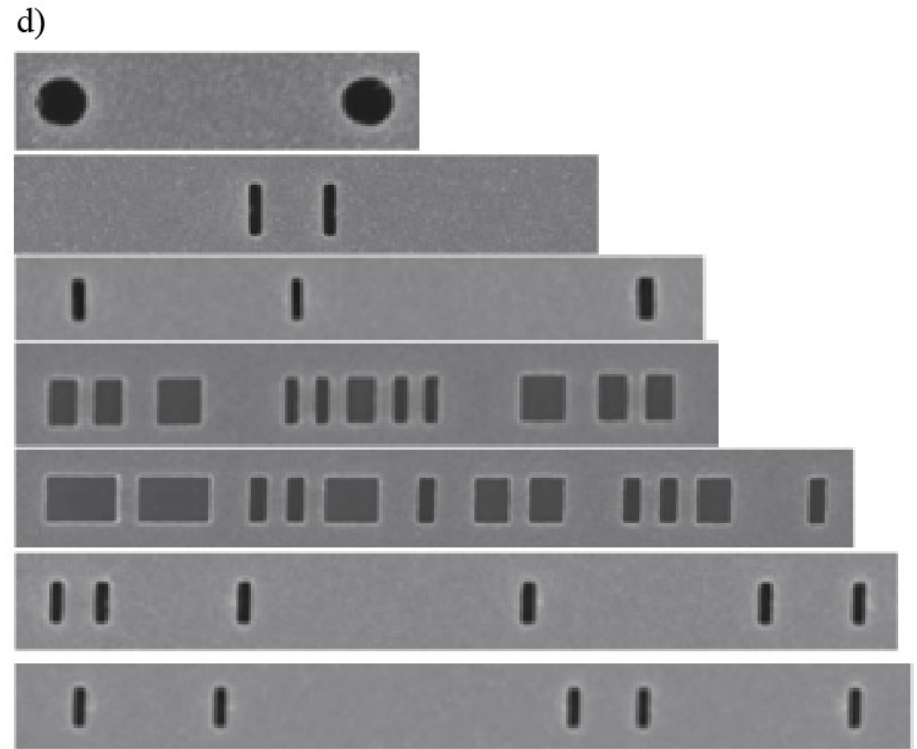
$$Z_R = 4\pi r_0^2/\lambda = 114 \text{ m @ } 10.3 \text{ keV}$$

P. Emma et al., First lasing and operation of an ångström-wavelength free-electronlaser, *Nature Photonics* **4**, 64 (2010); Proceed. PAC 2009; first lasing April 2009.

Measuring spatial coherence at LCLS



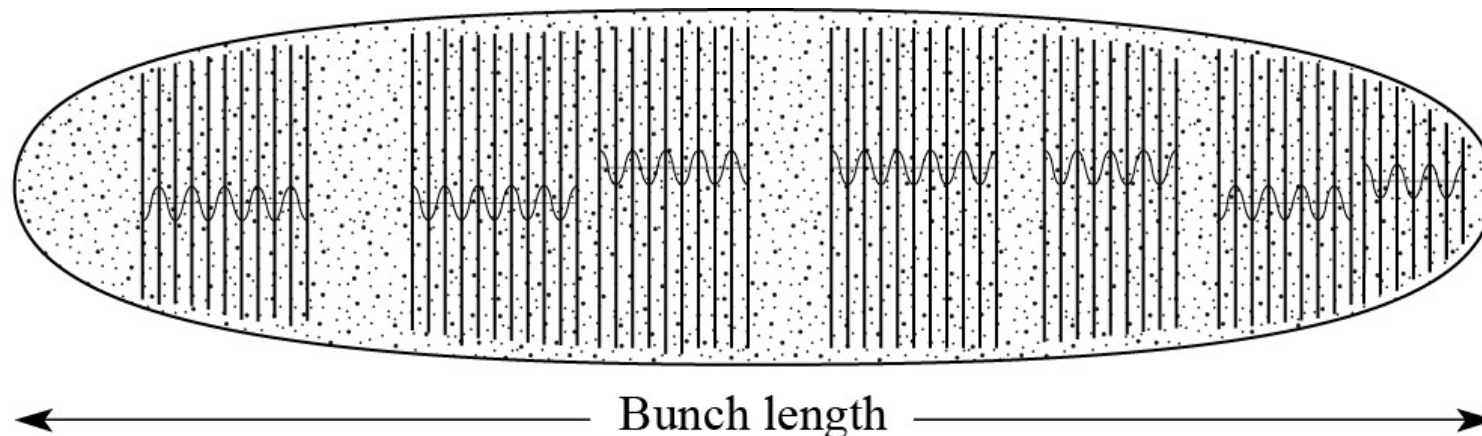
LCLS, 780 eV, 300 fsec, $\frac{1}{4}$ nC, 1mJ/pulse
78% energy in TEM_{00} mode



FEL_Ch6_F12_forSlides.ai

Courtesy of I. Vartanyants (DESY) and A. Sakdinawat (SLAC); PRL **107**, 144801 (30Sept2011)

The coherence properties of Free Electron Lasers

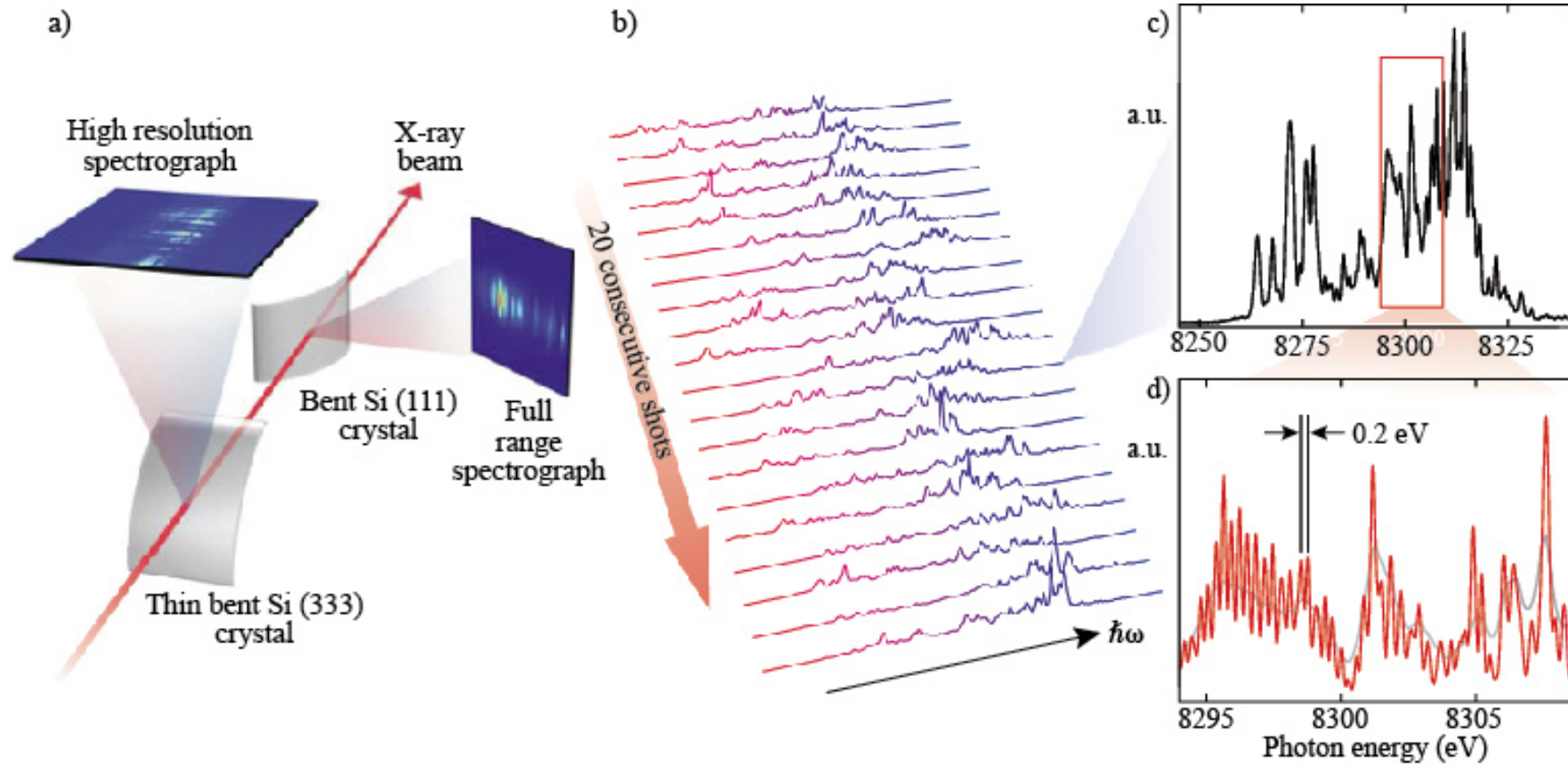


While “SASE” FEL radiation is to a large extent spatially coherent, the electron bunch length can support many independent coherent micro-bunches, thus temporal coherence is limited and spectral bandwidth relatively broad. This is a natural property of the “Self Amplified Spontaneous Emission” (SASE) FEL process. Coherent seeding with a sufficiently intense laser pulse seeks to overcome this limitation, with future plans largely centered on high harmonics of visible/IR lasers of higher power than presently available. Filtered “self-seeding” is also under consideration.

(LCLS: $2.35 \sigma_z = 21 \mu\text{m}$ FWHM vs. $\sim 400 \times .15 \text{ nm} \approx 0.06 \mu\text{m}$)

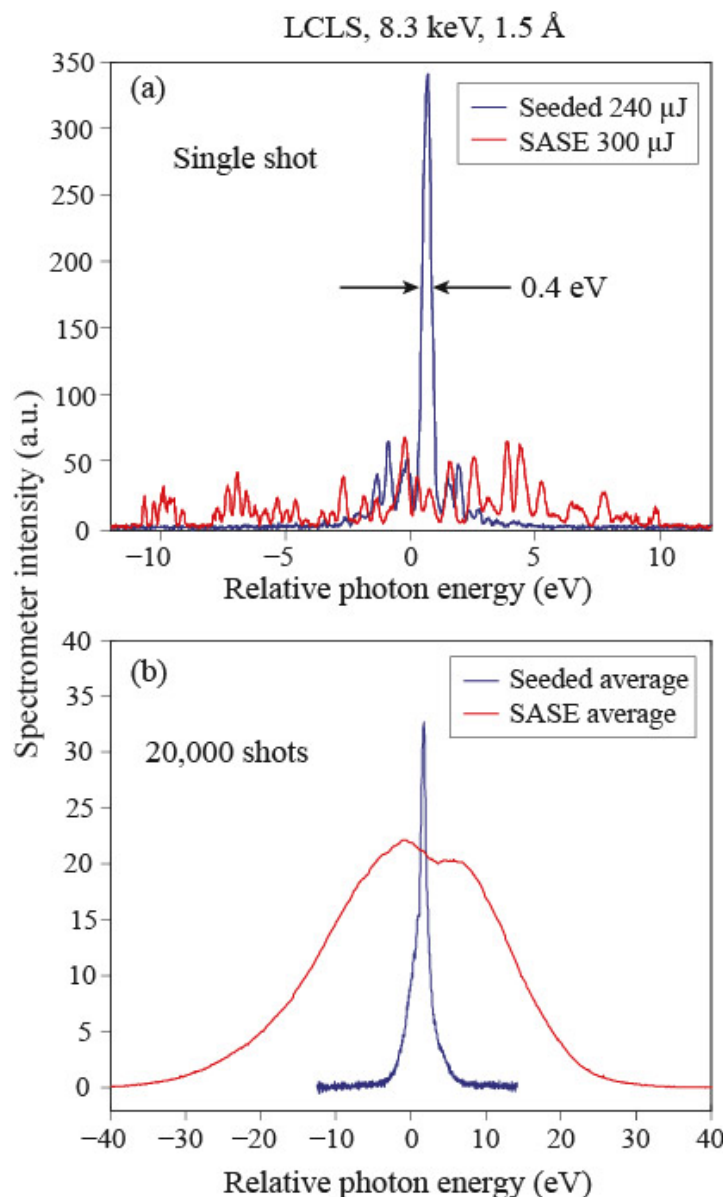
(\therefore Can support 300+ microbunches, depending on bunch/pulse duration)

Measuring the SASE spectrum at LCLS



Courtesy of D. Zhu, J. Hastings, W. Feng, (SLAC); APL **101**, 034103 (July 2012)

Reduced LCLS spectra, thus longer coherence length, with “self-seeding”



$$\frac{20 \text{ eV}}{0.4 \text{ eV}} = 50:1 \text{ reduction}$$

$$\frac{\Delta\lambda}{\lambda} = \frac{8 \text{ keV}}{0.4 \text{ eV}} = 2 \times 10^4$$

$$\ell_{\text{coh}} = \frac{\lambda^2}{2 \Delta\lambda} = \frac{2 \times 10^4}{2} \times 1.5 \text{ Å}$$

$$\ell_{\text{coh}} = 1.5 \text{ } \mu\text{m}$$

J. Amann, et al.,
Nature Photonics **6**, 693 (Oct 2012)



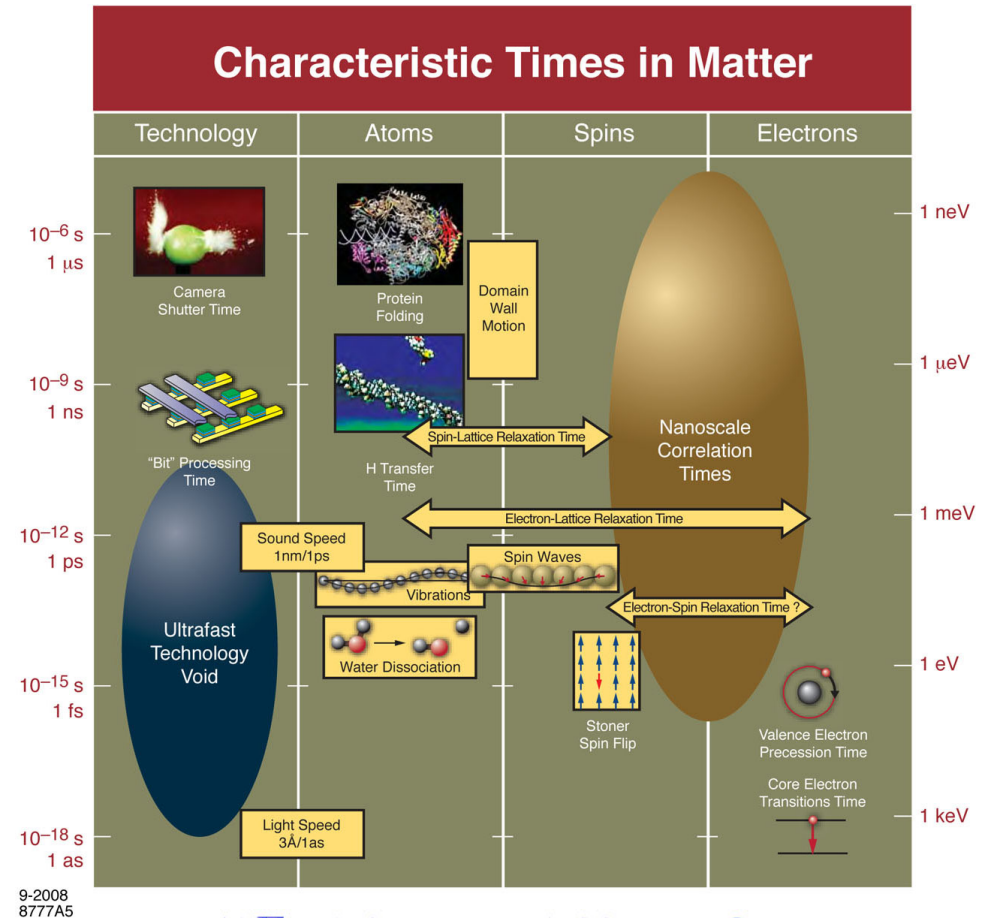
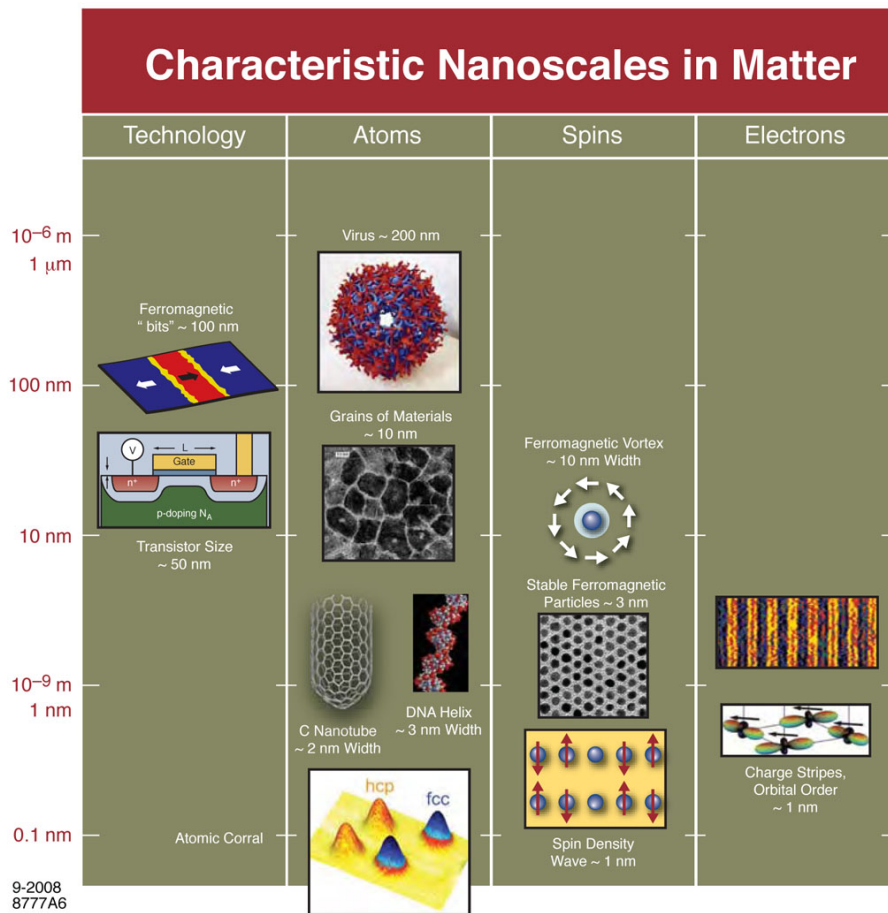
Typical FEL parameters

Parameters	Flash FEL (Hamburg 2005)	Fermi (Trieste, 2010)	LCLS (Stanford, 2009)	SACLA (Hyogo, 2011)	EU XFEL (Hamburg, 2015)
E_e	1.25 GeV	1.24 GeV	13.6 GeV	8 GeV	17.5 GeV
γ	2,450	2,300	26,600	15,700	35,000
\hat{I}	1.3 kA	300 A	3.4 kA	3 kA	5 kA
λ_u	27.3 mm	55 mm	30 mm	18 mm	35.6 mm
N	989	216	3733	4986	4000
L_u	27 m	14 m	112 m	90 m	200 m
$\hbar\omega$	30-300 eV (4-40 nm)	20-60 eV (20-60 nm)	250 eV - 12 keV (1-50 Å)	4.5-15 keV (0.8-2.8 Å)	4-12 keV (1-3 Å)
$\lambda/\Delta\lambda_{FWHM}$	100	1000	200-500	200-400	1000
$\Delta\tau_{FWHM}$	25 fsec	85 fsec	70 fsec	30 fsec	100 fsec
$\dot{\mathcal{F}}$ (ph/pulse)	3×10^{12}	5×10^{12}	2×10^{12}	5×10^{11}	10^{12}
rep rate	5 Hz	10 Hz	120 Hz	60 Hz	5 Hz/27 kHz
\hat{P}	1 GW	1 GW	25 GW	30 GW	20 GW
L	260 m	200 m	2 km	710 m	3.4 km
Polarization	linear	variable	linear	linear	variable (?)
Mode	SASE	Seeded (3 ω Ti: sapphire)	SASE	SASE	SASE

Flash II, Fermi II, SLS FEL, LCLS II,

FreeElectronLasersChart_June2014.ai

Probing matter on the scale of nanometers and femtoseconds



$$(\Delta E \cdot \Delta \tau)_{\text{FWHM}} = 1.82 \text{ eV} \cdot \text{fsec}$$

Science and Technology of Future Light Sources (Argonne, Brookhaven, LBNL and SLAC: Four lab report to DOE/Office of Science, Dec. 2008)



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2nd Edition in progress: new FEL, HHG, Coherence, and X-ray Imaging chapters



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www.coe.AST.berkeley.edu/sxr2009

www.coe.AST.berkeley.edu/srms

www.youtube.com

